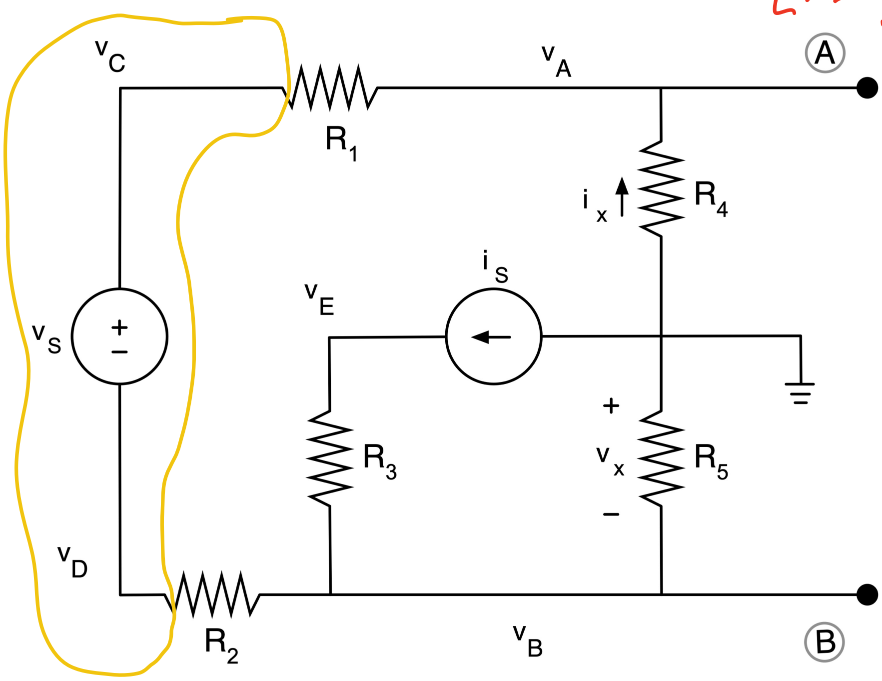
1.– Part I

To use node-voltage analysis, we must take care of the presence of the voltage source using one of the three methods discussed in class:

1) source transformation
2) grounding a node conveniently
3) supernode

We cannot use 1) because it involves modifying the circuit, something that the question statement says we cannot do. Re 2), ground has already been chosen, and <u>not</u> in a convenient way to take care of the voltage source. Therefore, we have to use a supernode. This supernode is the result of combining nodes C and D.

[+2 points]



The equation defining the supernode is

$$V_C - V_D = V_S$$ [+0.5 point]

Next, we write KCL for the supernode

$$G_1(V_C - V_A) + G_2(V_D - V_B) = 0$$ [+1 point]

(where we are using $G_i = \frac{1}{R_i}$ for convenience)

We also have to write KCL for nodes A, B, and E

KCL @ A:

$$G_1(V_A - V_C) + G_4 V_A = 0$$ [+0.5 point]

KCL @ B

$$G_5 V_B + G_3(V_B - V_E) + G_2(V_B - V_D) = 0$$ [+0.5 point]

KCL @ E

$$G_3(V_E - V_B) = i_S$$ [+0.5 point]

With these, we have 5 eqs in 5 unknowns V_A, V_B, V_C, V_D, V_E.

In matrix form either [+1 point]

$$\begin{pmatrix} G_1+G_4 & 0 & -G_1 & 0 & 0 \\ 0 & G_2+G_3+G_5 & 0 & -G_2 & -G_3 \\ 0 & 0 & 1 & -1 & 0 \\ -G_1 & -G_2 & G_1 & G_2 & 0 \\ 0 & -G_3 & 0 & 0 & G_3 \end{pmatrix} \begin{pmatrix} V_A \\ V_B \\ V_C \\ V_D \\ V_E \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ V_S \\ 0 \\ i_S \end{pmatrix}$$

Part II

In terms of the node voltages, we have

$$v_x = -V_B \qquad [+1 \text{ point}]$$

$$i_x = G_4(-V_A) \qquad [+1 \text{ point}]$$

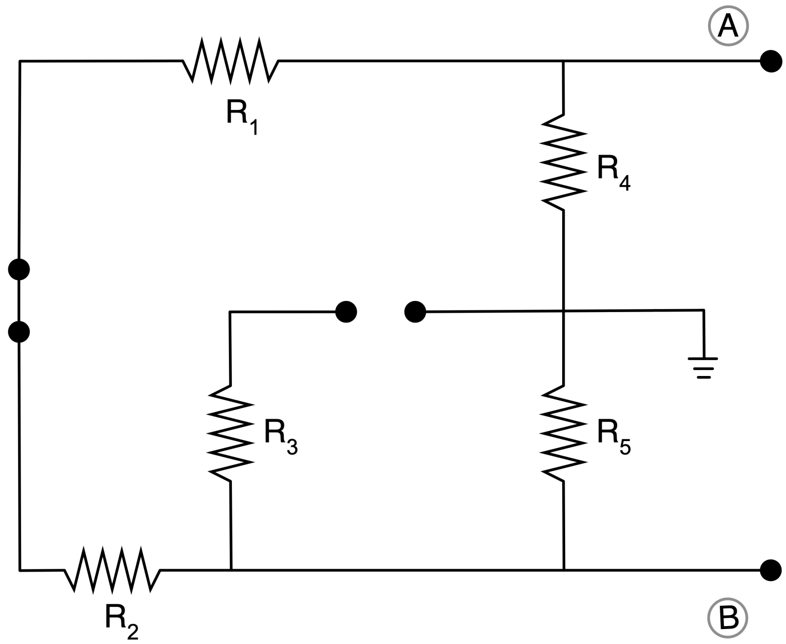
Part III

If source transformation was allowed, we could have transformed the voltage source (which is a problem for node voltage analysis) in series with R_1 (or with R_2) into a current source in parallel with the corresponding resistor. This would also have decreased the number of nodes by one, hence simplifying the setting up of the NV equations.

[+2 points]

2._ Part I

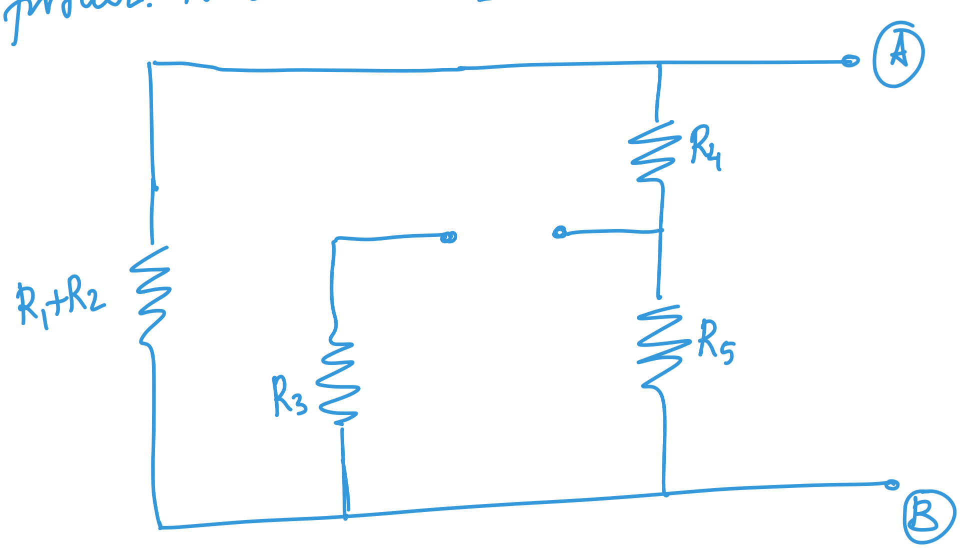
We turn off all the sources in the circuit and obtain the circuit below



[+0.5 point]

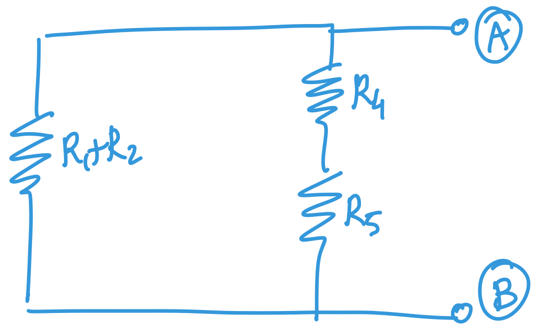
where the voltage source gets replaced by a short circuit, and the current source by an open circuit. [+0.5 point]

Next, we use association of resistors to simplify it further. Note that R_1 and R_2 are in series, so



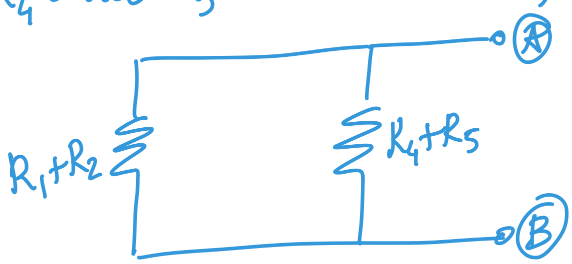
[+0.5 point]

Moreover, there is no current going through R_3 (because of the open circuit), so it is as if that resistor was not there.



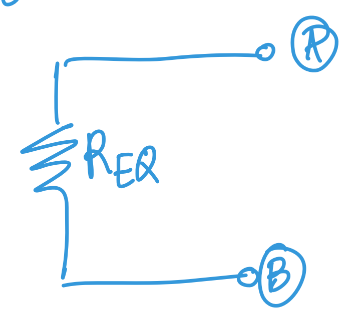
[+0.5 point]

R_4 and R_5 are in series, so we simplify as



[+0.5 point]

Finally, the two remaining resistors are in parallel, so



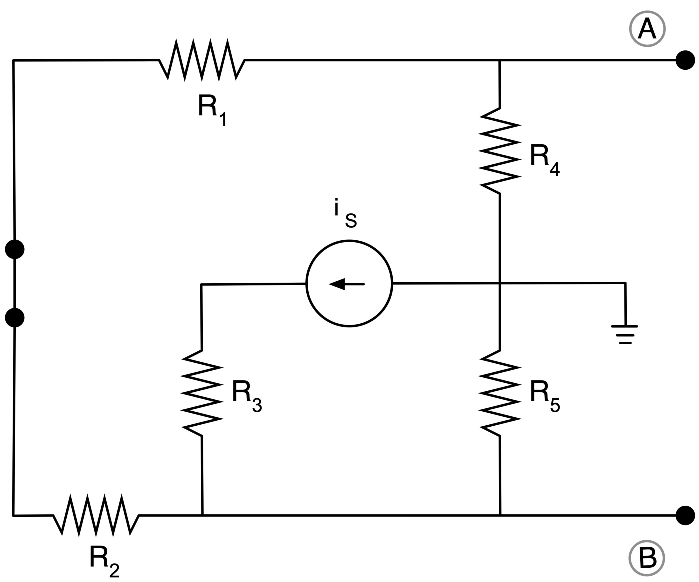
$$R_{EQ} = (R_1+R_2) \parallel (R_4+R_5)$$

$$= \frac{(R_1+R_2)\cdot(R_4+R_5)}{R_1+R_2+R_4+R_5}$$

[+0.5 point]

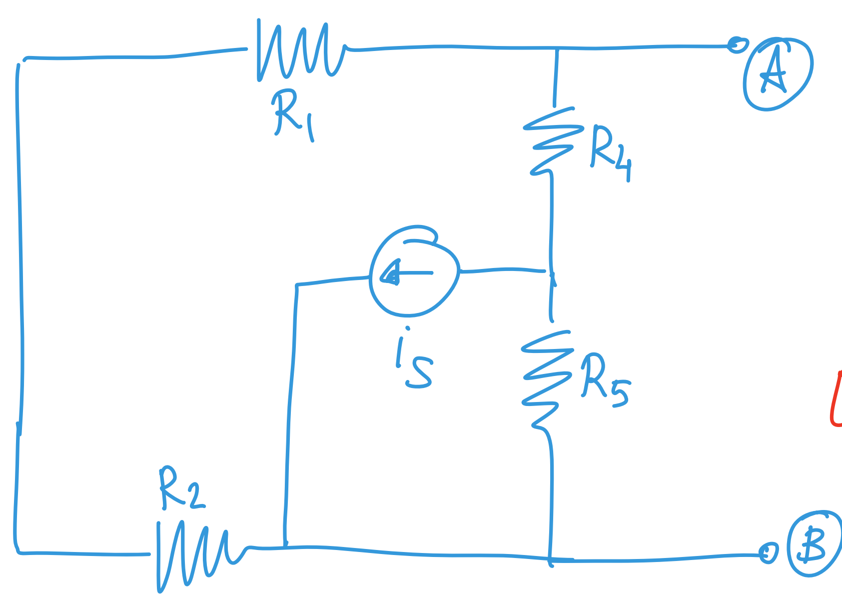
Part II

We turn off the voltage source, substituting it by a short circuit as



[+0.5 point]

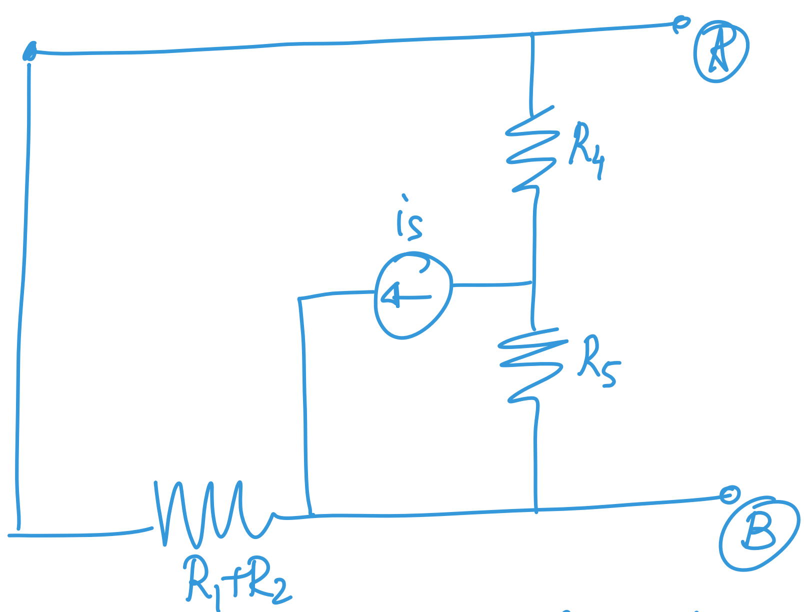
Since the resistor R_3 is in series with a current source, from the point of view of the rest of the circuit (and, in particular, to compute the open circuit voltage), this is equivalent to



[+0.5 point]

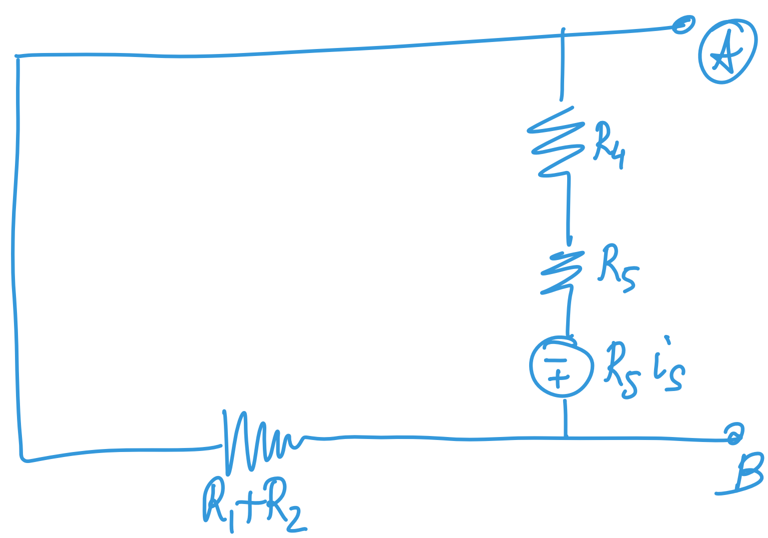
Now we observe that R_1 and R_2 are in series,

so



[+0.5 point]

Next, we use source transformation and turn the current source in parallel w/ R_5 into a voltage source in series with R_5.



[+0.5 point]

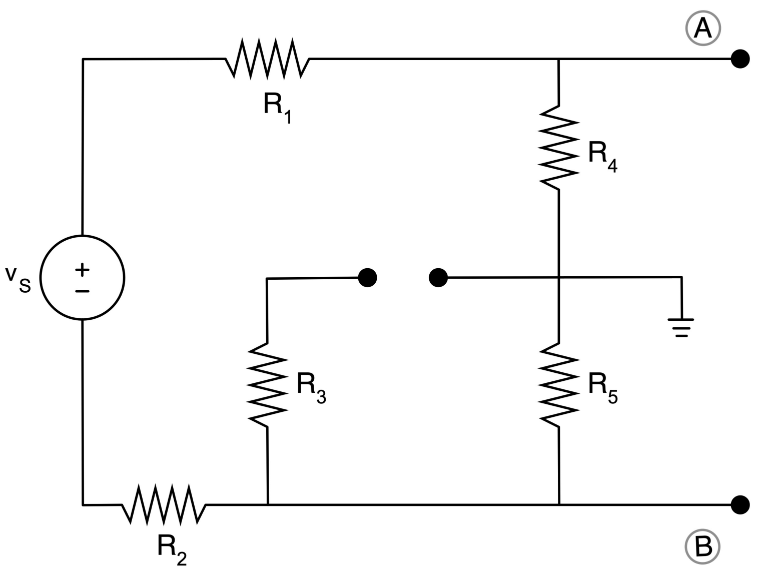
Finally, we see that the open circuit voltage is the voltage drop seen by the resistor R_1+R_2, which can be computed by voltage division

$$(V_{AB})_1 = V_{R_1+R_2} = \frac{R_1+R_2}{R_1+R_2+R_4+R_5}(-R_5 i_s)$$

[+1 point]

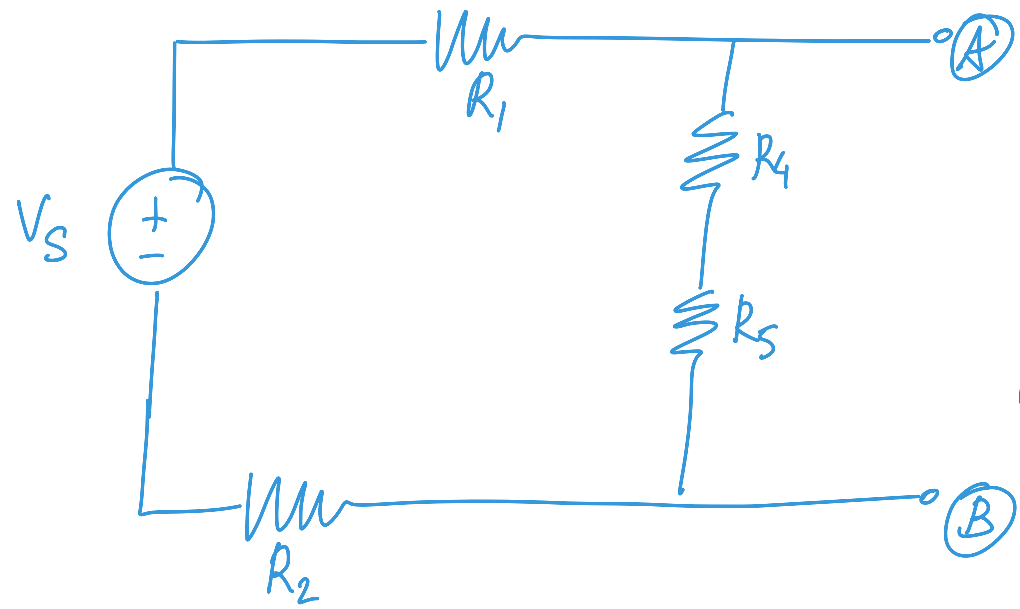
Part III

We turn off the current source, substituting it by an open circuit as



[+0.5 point]

R_3 does not play any role because of the lack of current going through it, so we redraw as



[+0.5 point]

We see that the open circuit voltage is the sum of the voltage drops seen by R_4 and R_5. This we can find using voltage division

$$(V_{AB})_2 = \frac{R_4 + R_5}{R_1 + R_2 + R_4 + R_5} V_S$$

[+1 point]

Part IV

By superposition, the open circuit voltage as seen from terminals A and B is the sum of the answers of Part II (voltage source off) and Part III (current source off).

[+1 point]

Therefore,

$$V_T = V_{OC} = (V_{AB})_1 + (V_{AB})_2 =$$

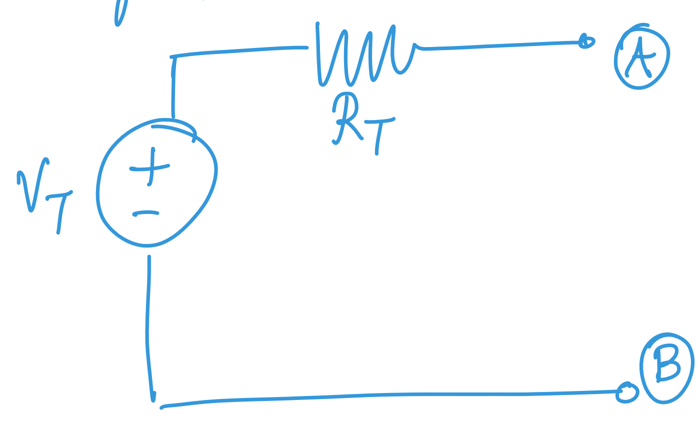
$$= \frac{R_1+R_2}{R_1+R_2+R_4+R_5}(-R_5 i_S) + \frac{R_4+R_5}{R_1+R_2+R_4+R_5} V_S$$

$$= \frac{(R_4+R_5)V_S - (R_1+R_2)R_5 i_S}{R_1+R_2+R_4+R_5}$$ [+0.5 point]

From Part I,

$$R_T = R_{EQ} = \frac{(R_1+R_2)(R_4+R_5)}{R_1+R_2+R_4+R_5}$$ [+0.5 point]

Therefore, the Thevenin equivalent is



Part V

The answer would be the same if the resistor R_3 was not present. This is because the current source in series with that resistor is equivalent to the current source by itself. This is also apparent in the response for V_T and R_T given in Part IV, where the conductance G_3 does not show up.

[+1 extra point]