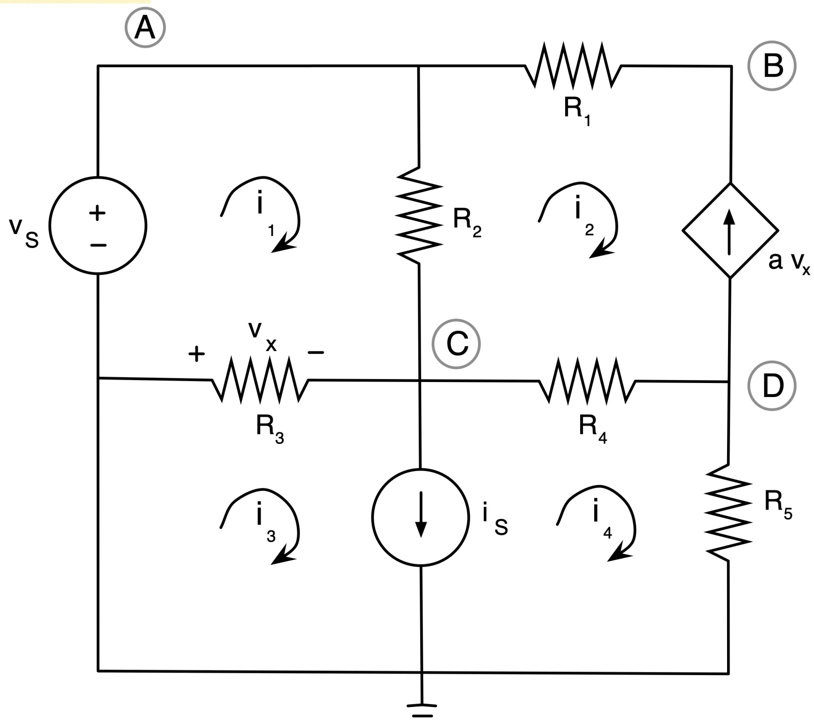
1.- Part I



Looking at the circuit, we observe the presence of 2 current sources, which are problems we need to deal with to use mesh current analysis. One of them only belongs to one mesh (method 2), but the other belongs to two, so we will use a supermesh for it (method 3).

So, method 2 for the current source on the right yields

$$i_2 = -a v_x$$ [+1 point]

The supermesh equation is

$$i_3 - i_4 = i_S$$ [+1 point]

And KVL for the supermesh reads

$$R_3(i_3-i_1)+R_4(i_4-i_2)+R_5 i_4=0$$ [+1 point]

We also write KVL for mesh 1 as

$$R_2(i_1-i_2)+R_3(i_1-i_3)-V_S=0$$ [+1 point]

Finally, we need to account for the presence of a dependent source. Looking at the circuit, we see that

$$V_x=R_3(i_3-i_1)$$ [+1 point]

Our discussion above yields a total of 5 eqs. in 5 unknowns, i_1, i_2, i_3, i_4, V_x. [+1 point]

In matrix form, this can be expressed as

$$\begin{pmatrix} 0 & 1 & 0 & 0 & a \\ 0 & 0 & 1 & -1 & 0 \\ -R_3 & -R_4 & R_3 & R_4+R_5 & 0 \\ R_2+R_3 & -R_2 & -R_3 & 0 & 0 \\ R_3 & 0 & -R_3 & 0 & 1 \end{pmatrix}\begin{pmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ V_x \end{pmatrix}=\begin{pmatrix} 0 \\ i_S \\ 0 \\ V_S \\ 0 \end{pmatrix}$$

Part II

$$V_A = V_S$$ [+1 point]

$$V_B = R_1(-i_2) + V_A = -Ri_2 + V_S$$ [+1 point]

$$V_C = R_3(i_1 - i_3)$$ [+1 point]

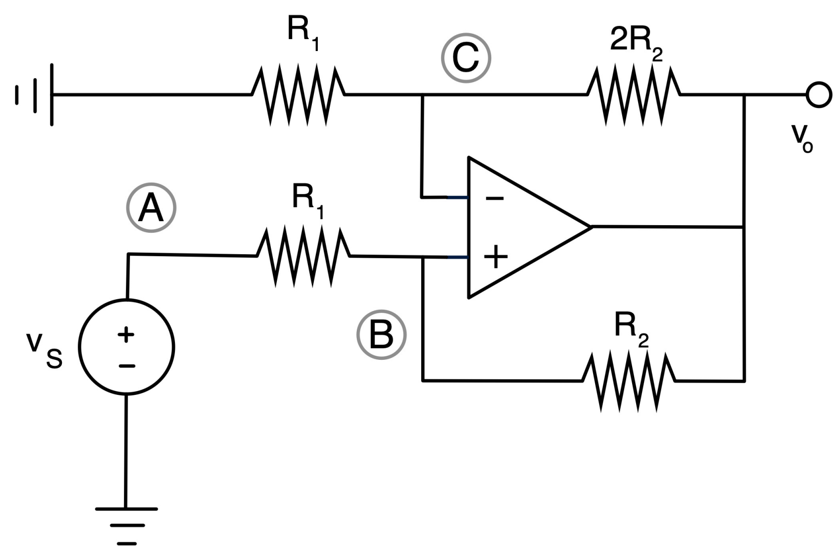
$$V_D = R_5\, i_4$$ [+1 point]

Part III

No, the mesh currents will not change. This is because the resistor R_1 is in series with a current source, and we know that, from the point of view of the rest of the circuit, this is equivalent to the current source by itself. [+1 extra point]

Another way to justify this is to realize that the value of R_1 does not affect the equations in Part I.

2.-



Part I

As instructed, we use nodal analysis to figure out the output voltage.

We know $V_A = V_S$. [+1 point]

KCL at node B (with $i_p = 0$),

$$\frac{1}{R_1}(V_B - V_A) + \frac{1}{R_2}(V_B - V_o) = 0$$ [+1 point]

KCL at node C (with $i_n = 0$),

$$\frac{1}{R_1}(V_C - 0) + \frac{1}{2R_2}(V_C - V_o) = 0$$ [+1 point]

Ideal conditions mean that

$$V_B = V_C$$ [+1 point]

We have 3 eqs. in 3 unknowns V_B, V_C, V_0, so we can solve. From the 1st eq,

$$\frac{1}{R_2} V_0 = \left(\frac{1}{R_1} + \frac{1}{R_2}\right) V_B - \frac{1}{R_1} V_S$$

$$V_0 = \left(1 + \frac{R_2}{R_1}\right) V_B - \frac{R_2}{R_1} V_S$$

From the 2nd eq.,

$$\frac{1}{2R_2} V_0 = \left(\frac{1}{R_1} + \frac{1}{2R_2}\right) V_C$$

$$V_0 = \left(1 + 2\frac{R_2}{R_1}\right) V_C$$

Therefore

$$\left(1 + \frac{R_2}{R_1}\right) V_B - \frac{R_2}{R_1} V_S = \left(1 + \frac{2R_2}{R_1}\right) V_B$$

Simplifying,

$$-\frac{R_2}{R_1} V_S = \frac{R_2}{R_1} V_B \Rightarrow V_B = -V_S$$

Therefore,

$$V_0 = -\left(1 + \frac{2R_2}{R_1}\right) V_S \qquad [+1 \text{ point}]$$

And the correct answer is the last one.

Part II

With the values provided, we have

$$-10 \leq V_0 = -(1+1)V_S \leq 20 \qquad [+1 \text{ point}]$$

$$-10 \leq -2V_S \leq 20$$

$$5 \geq V_S \geq -10$$

So the input V_S must be between $-10V$ and $5V$ for linear operation. [+1 point]

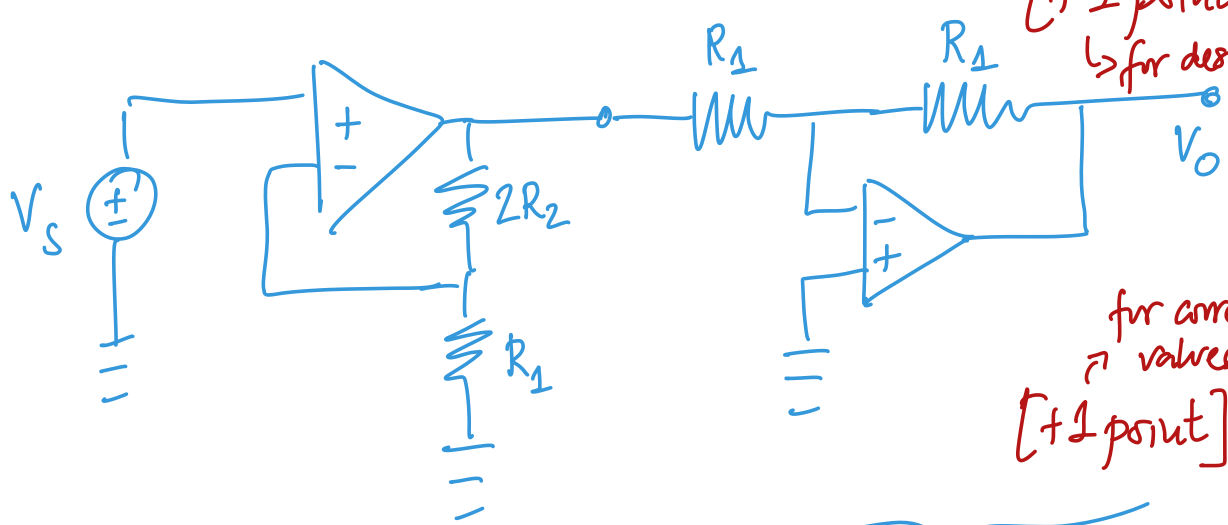
Part III

Note that we can write

$$V_0 = -\left(1 + \frac{2R_2}{R_1}\right)V_S = (-1)\cdot\left(\frac{R_1 + 2R_2}{R_1}\right)V_S$$

This can be realized with a non-inverting op-amp and an inverting op-amp, as follows.

[+1 point]
↳ for design



for correct values
[+1 point]

non-inverting op-amp

gain: $\dfrac{2R_2+R_1}{R_1}$

inverting op-amp

gain: $-\dfrac{R_1}{R_1} = -1$

Note that the inverting op-amp does not load the non-inverting op-amp because of the zero output resistance of the latter. Therefore

[+1 point]

$$V_o = (-1)\cdot\left(\frac{2R_2+R_1}{R_1}\right)V_s\,.$$