Equivalent Circuits & Reduction (T&R Chap 2,3)

Circuit Equivalence

Two circuits are equivalent if they have the same *i-v* characteristics at a specified pair of terminals

Terminal = external connection to two nodes = port

Our aim is to simplify analysis replacing complicated subcircuits by simpler equivalent circuits

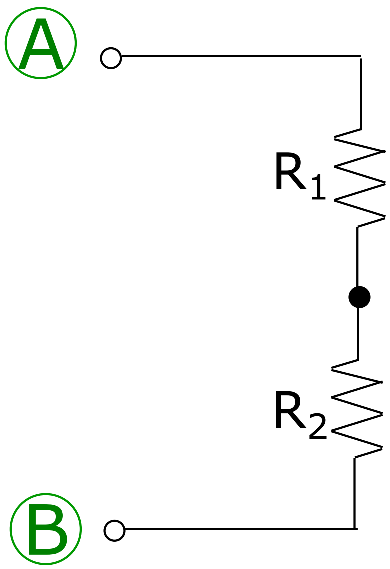
Example

Association of resistors

Equivalent Circuits & Reduction (T&R Chap 2,3)

Resistors in Series

$$R_{EQ} = R_1 + R_2$$



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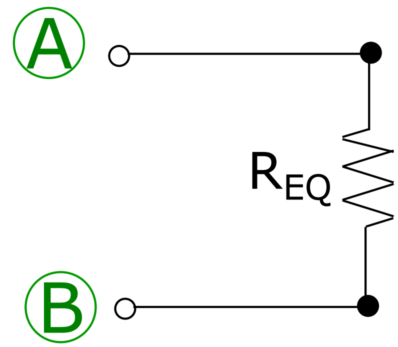
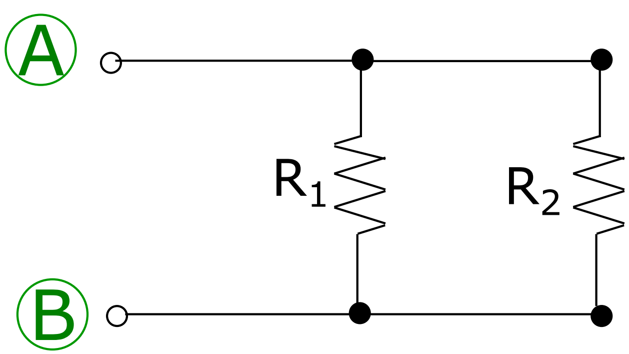


Resistors in Parallel

$$R_{EQ} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_1 R_2}{R_1 + R_2}$$



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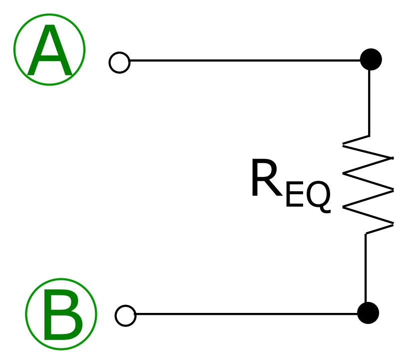
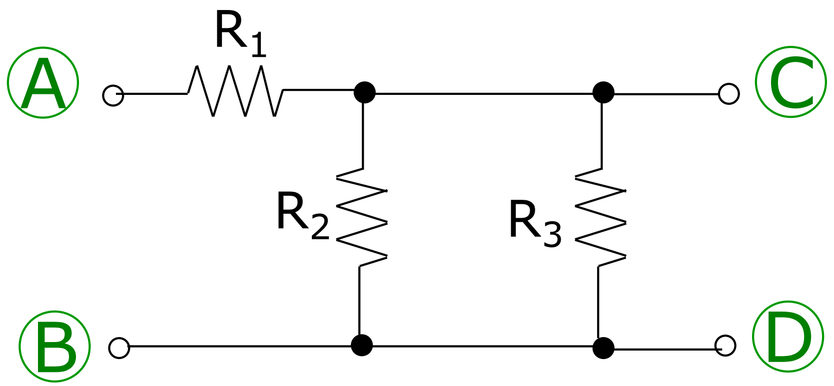


Example 2-11 (T&R, 5th ed, p. 34)

Consider the circuit



Compute equivalent ccts from AB and from CD

$$R_{EQ_{CD}} = R_2 \parallel R_3 = \left(\frac{1}{R_2} + \frac{1}{R_3}\right)^{-1} = \frac{R_2 R_3}{R_2 + R_3}$$

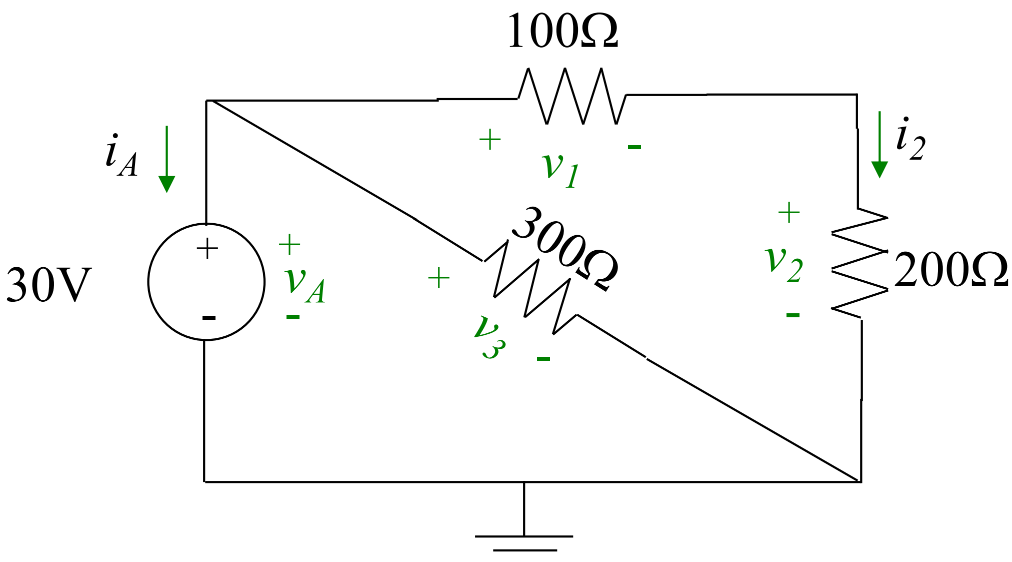
$$R_{EQ_{AB}} = R_1 + R_2 \parallel R_3 = R_1 + \frac{R_2 R_3}{R_2 + R_3}$$

Example 2-10 (T&R, 5th ed, p. 31) Revisited

Can you find i_A?

All you need to know is

$v_A = 30V$



Example 2-10 (T&R, 5th ed, p. 31) Revisited

Can you find i_A?

All you need to know is

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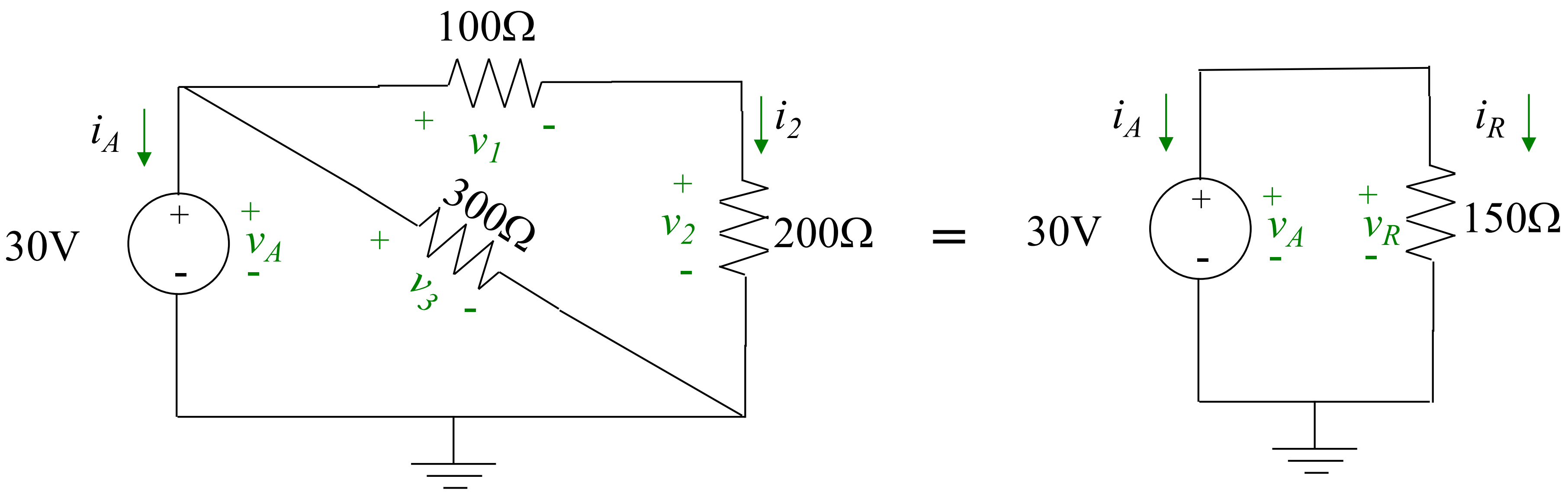
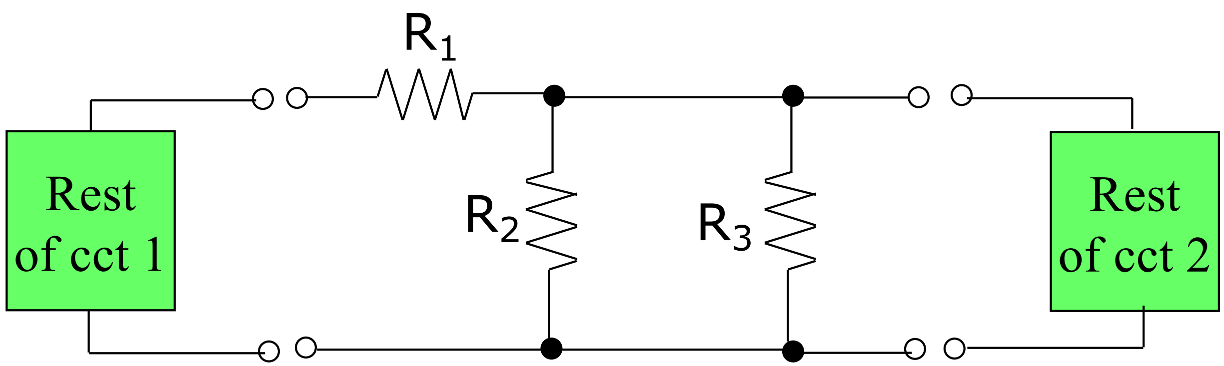


Answer

$$i_A = -i_R = -30/150 = -200\ mA;$$

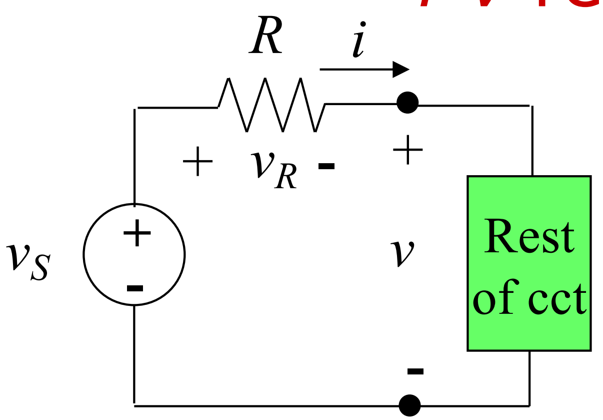
Equivalent ccts

- Equivalent ccts for resistive networks are familiar reductions of parallel and series connections
  - The equivalent cct depends on the port
- From an external view the cct could be replaced by its equivalent
  - The internal cct variables are now unavailable
    - How would you compute them?
- Could we substitute for the cct below?

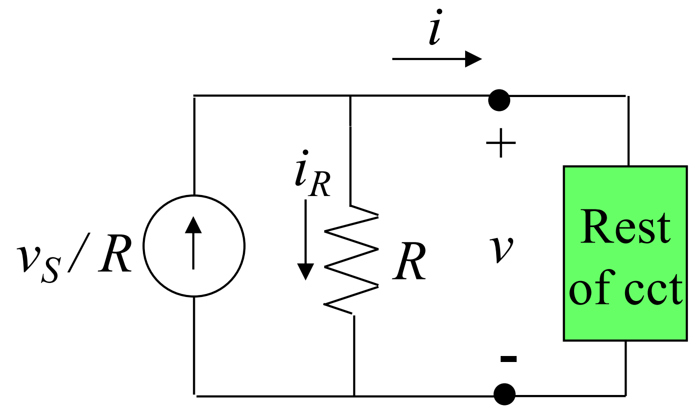


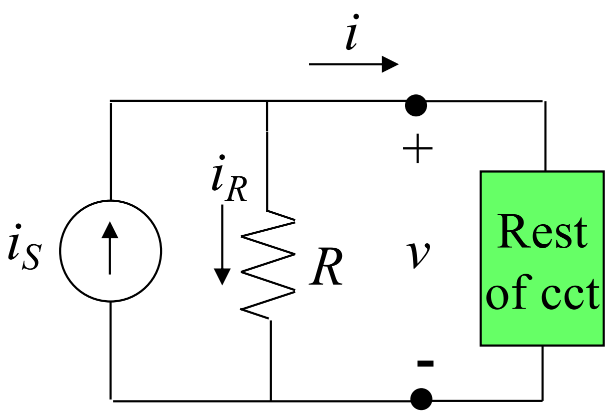
Equivalent sources

i-v relationships determine equivalence

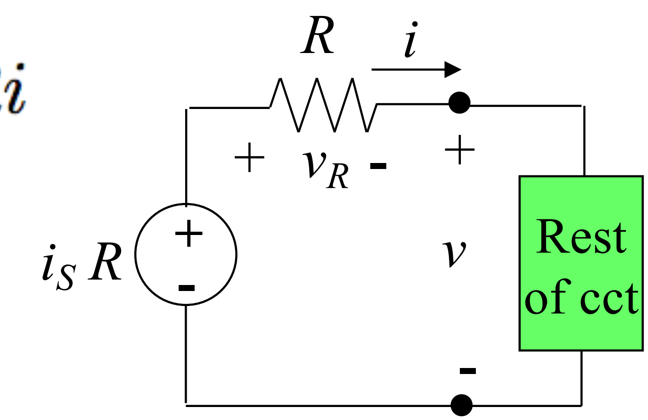


$$i = \frac{v_S}{R} - \frac{v}{R}$$

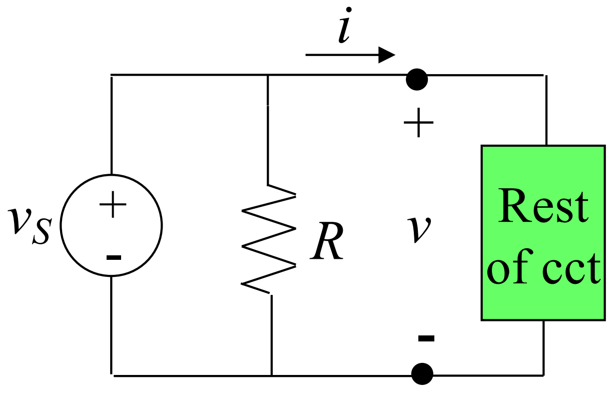




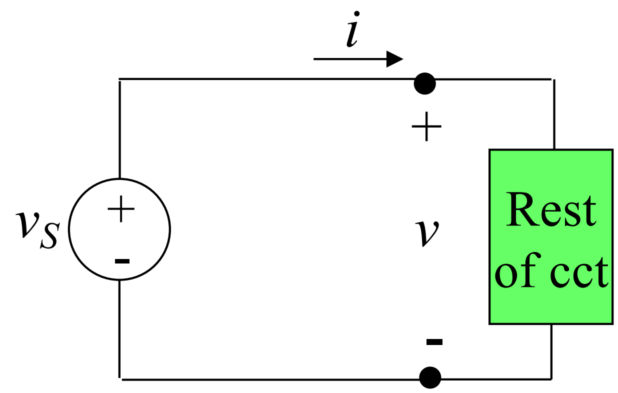
$$v = (i_S R) - Ri$$

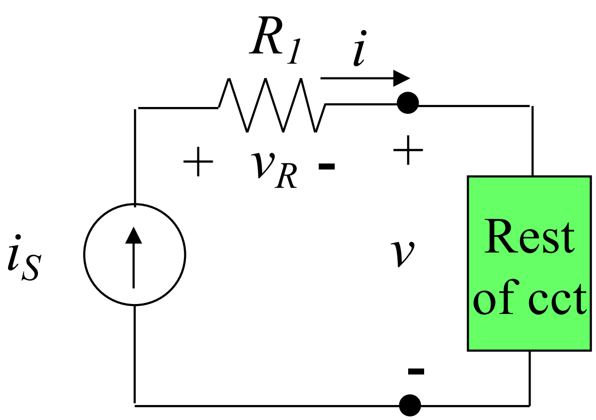


Equivalent Sources (cntd)

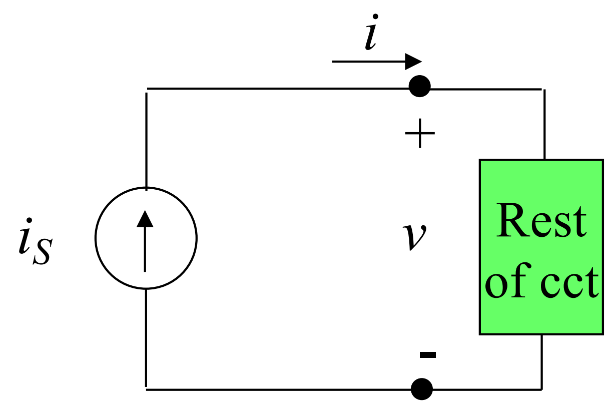


$$v = v_S$$

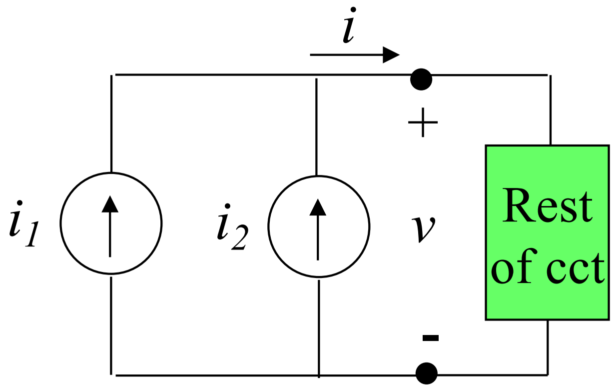




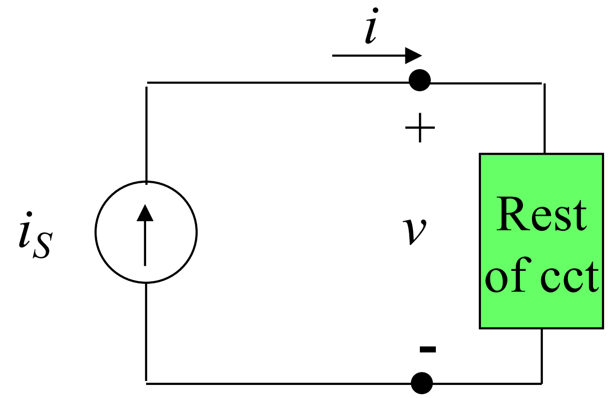
$$i = i_S$$

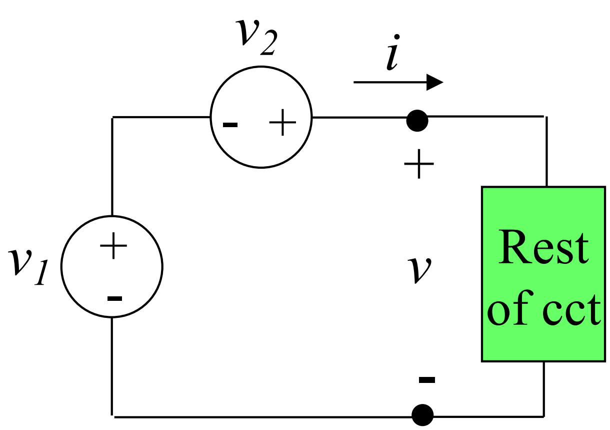


Equivalent Sources (cntd)

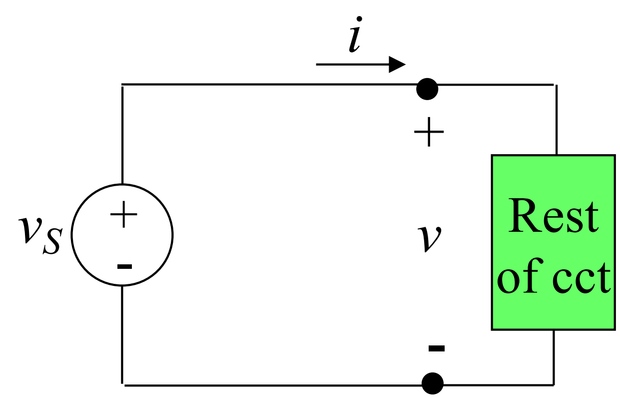


$$i_S = i_1 + i_2$$

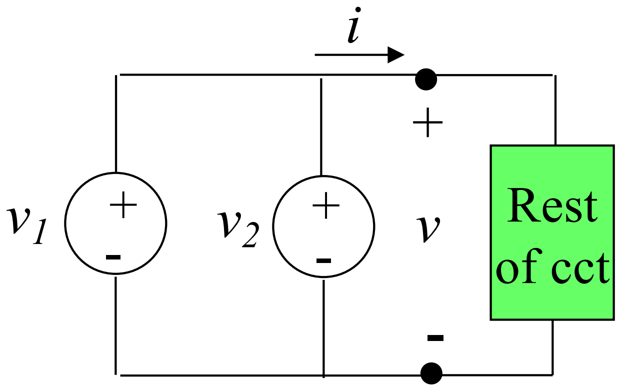




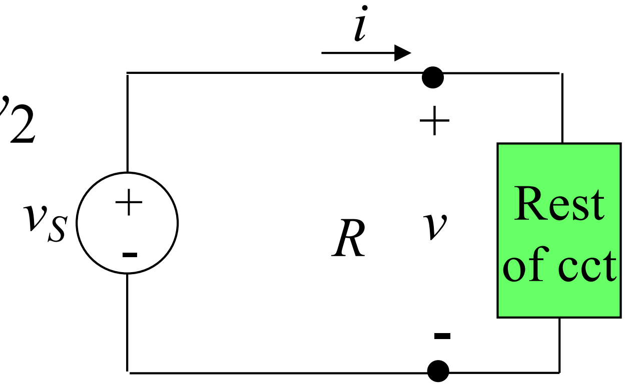
$$v_S = v_1 + v_2$$

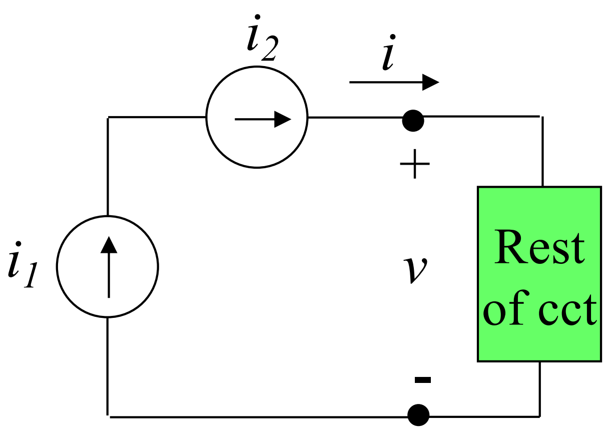


Equivalent Sources (cntd)

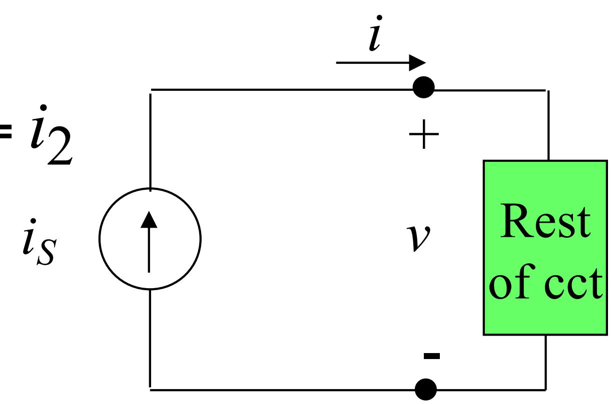


Only works if $v_1 = v_2$
then $v_S = v_1 = v_2$



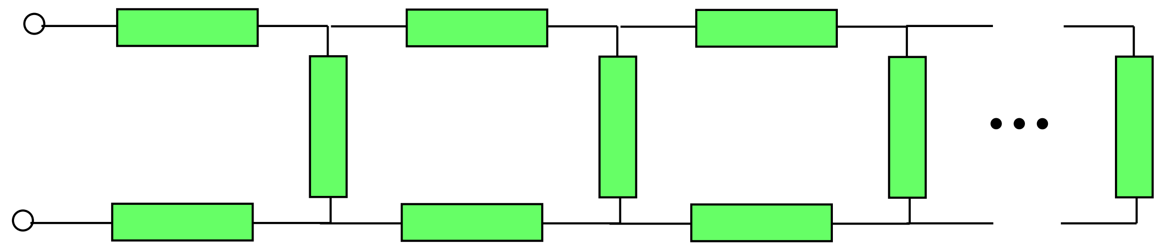


Only works if $i_1 = i_2$
then $i_S = i_1 = i_2$



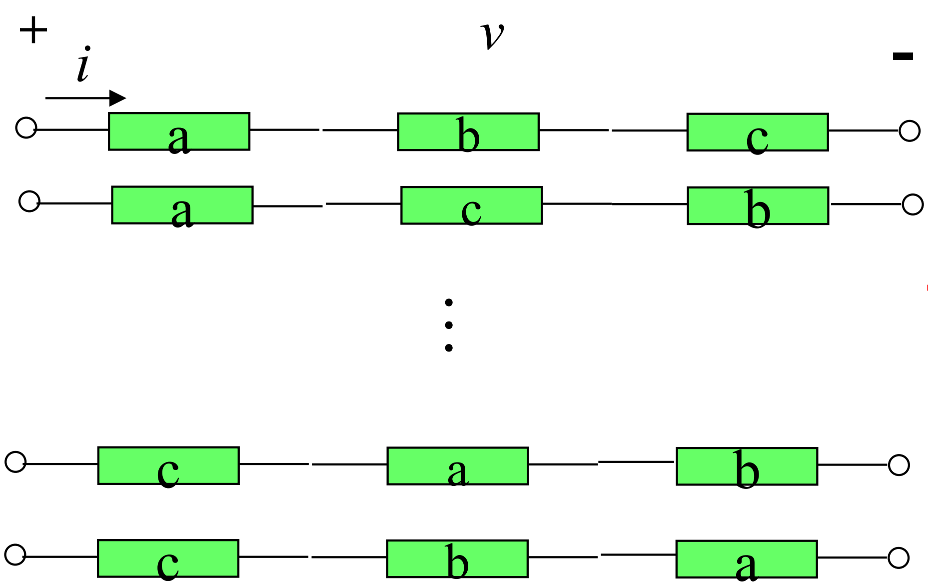
Circuit Reduction

For ladder networks



Reduce complexity by successively replacing elements by their equivalents

What happens with three elements in series or in parallel?

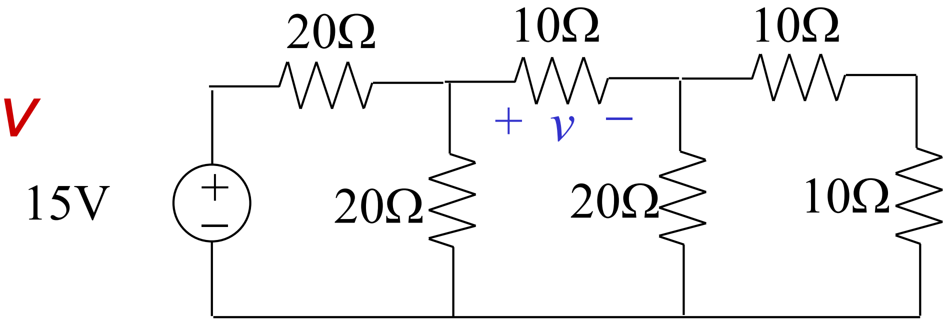


They are all equivalent

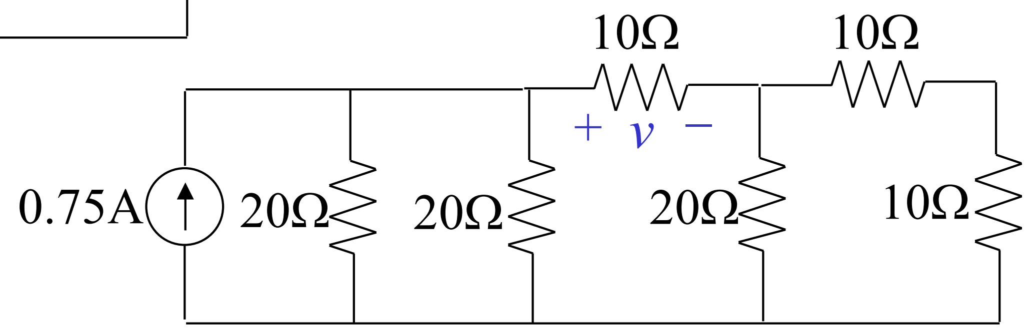
We can commute elements

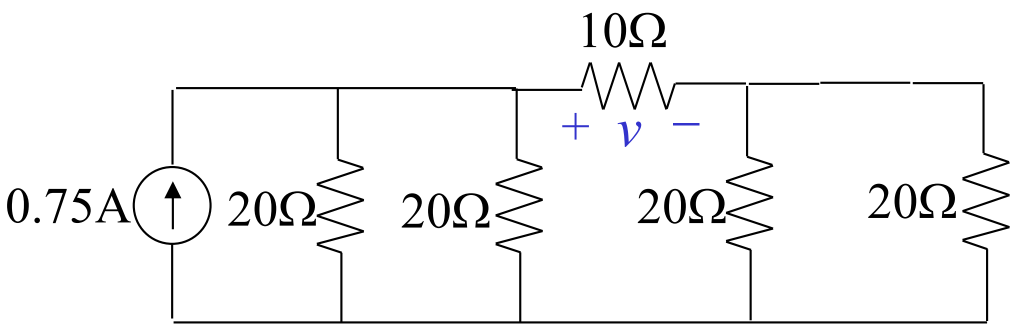
T&R, 5th Ed, Example 2-22 p 49

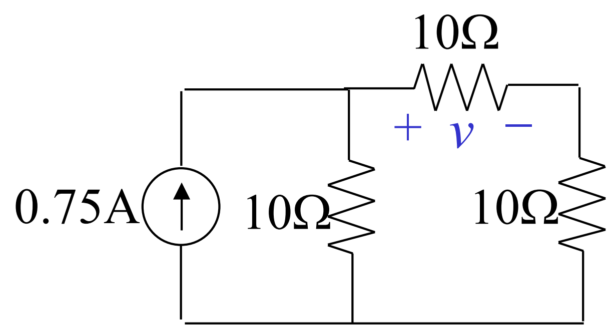
Find *v*

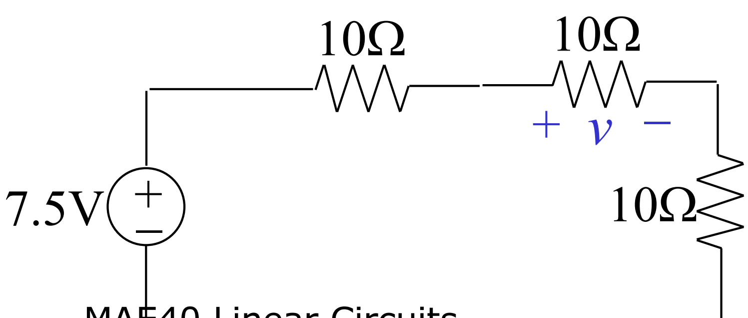


Reduce the left end



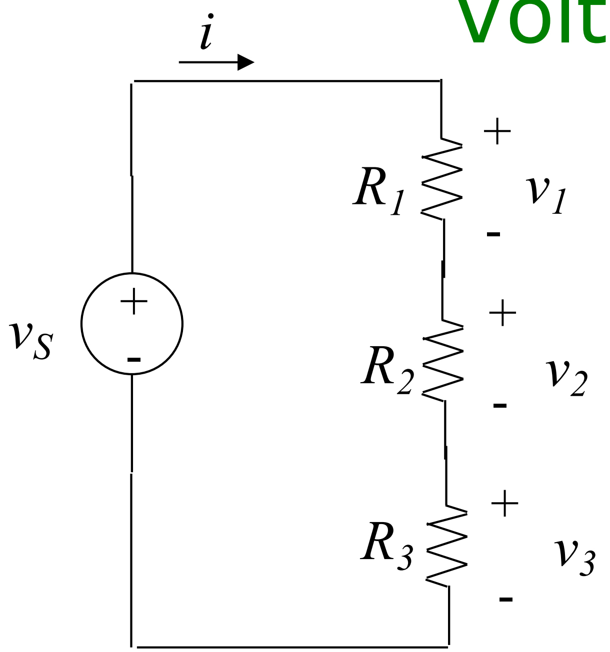






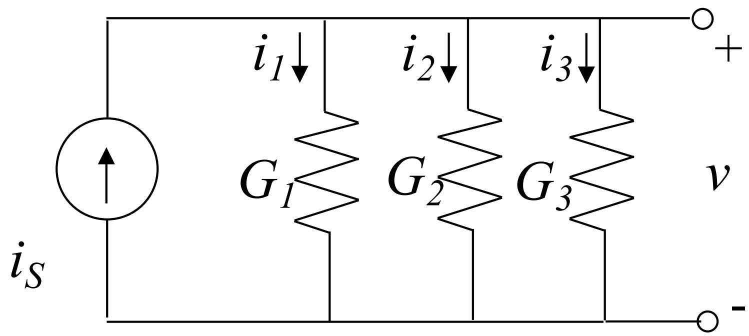
$v = 2.5V$

Voltage & Current Dividers



$$R_{total} = R_1 + R_2 + R_3$$

$$v_1 = \frac{R_1}{R_{total}} v_S; \quad v_2 = \frac{R_2}{R_{total}} v_S; \quad v_3 = \frac{R_3}{R_{total}} v_S$$

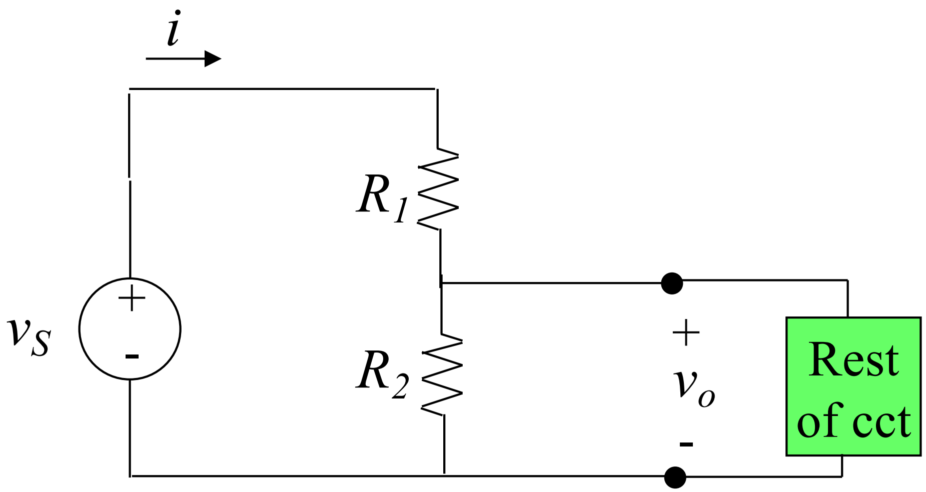


$$G_{total} = G_1 + G_2 + G_3$$

$$i_1 = \frac{G_1}{G_{total}} i_S; \quad i_2 = \frac{G_2}{G_{total}} i_S; \quad i_3 = \frac{G_3}{G_{total}} i_S$$

$$G_i = \frac{1}{R_i}$$

Voltage Dividers



Often we use a voltage divider to provide an input voltage to a cct element

When would this work?

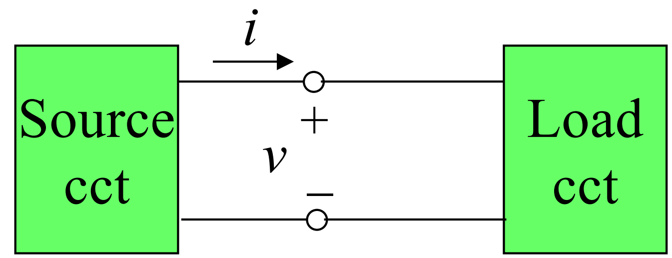
When the "rest of cct" does not draw much current compared to R_2

Why is this?

What is it asking of the equivalent of the rest of cct?

Note that this is a very common circuit used to "bias" a transistor to an operating voltage

Thévenin Equivalent Ccts



Thévenin's Theorem

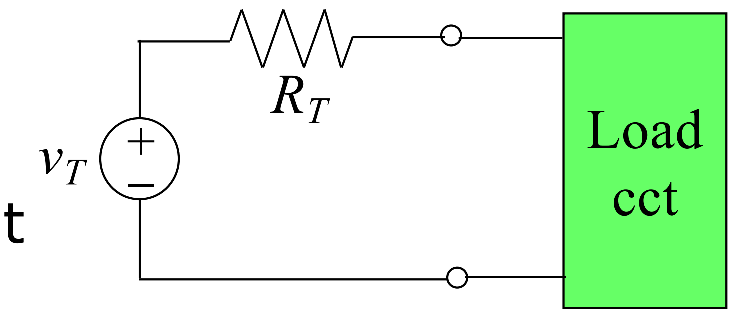
If the source cct in a two-terminal interface is linear, then the interface signals v and i do not change when the source cct is replaced by its Thévenin equivalent

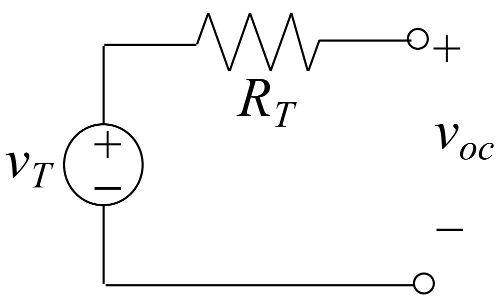
Note: nobody says the load must be linear!

Thévenin Equivalent Circuit

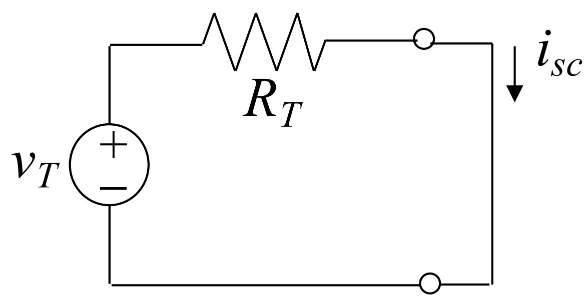
v_T is the open-cct voltage of source

R_T is evaluated from short-cct current





$$v_T = v_{oc}$$



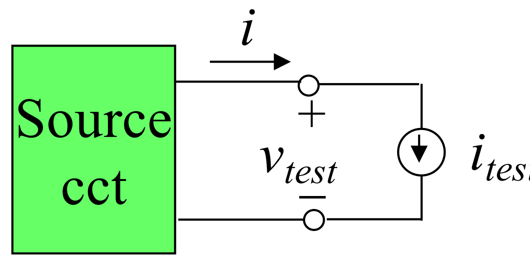
$$i_{sc} = \frac{v_T}{R_T} = \frac{v_{oc}}{R_T}$$

$$R_T = \frac{v_{oc}}{i_{sc}}$$

Thévenin's Theorem Proof

Linearity of the Source cct is the key – superposition

"Linear cct response to multiple sources is the sum of the responses to each source"



Hook up a test current source to cct

i_{test} yields voltage v_{test}

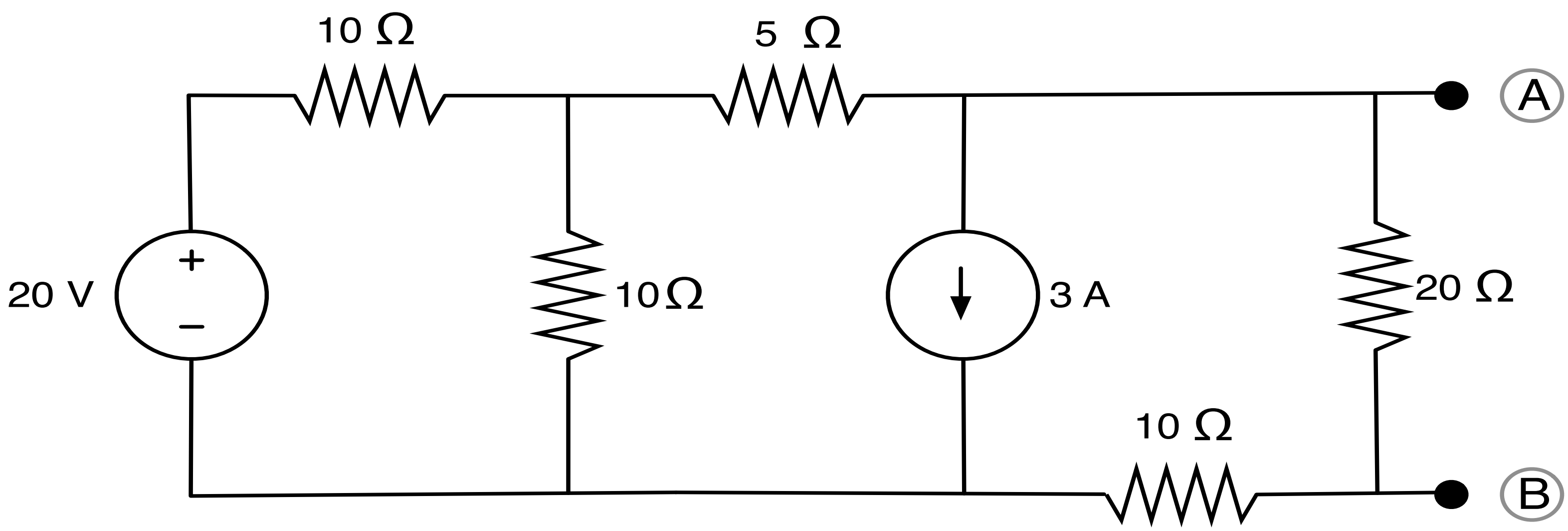
Part I, $i_{test,1}=0$ but i and v sources in Source cct left ON then $v_{test,1}=v_{oc}=v_T$

Part II, $i_{test,2}\neq 0$ and sources left OFF in Source cct then $v_{test,2}=-R_T i_{test,2}$

By linearity of the Source cct v_{test} is the sum of these parts for any choice of i_{test}

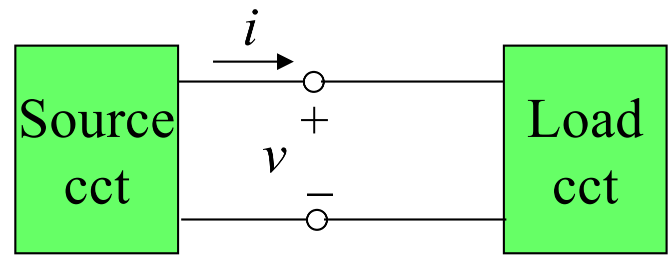
This establishes the i-v relationship for any load cct

Exercise from midterm, Fall 11



1. Turn off all the sources in the circuit and find the equivalent resistance as seen from terminals A-B **(10*Ω*)**
2. Find the Thévenin equivalent as seen from terminals A-B **(-10V)**
3. Find the power absorbed by a 40*Ω* resistor that is connected to terminals A-B **(1.6W)**

Norton Equivalent Ccts



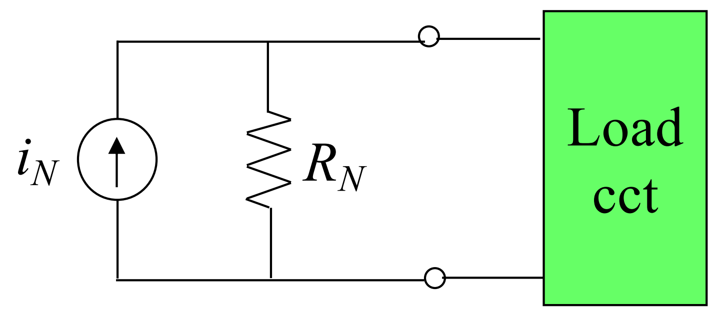
Norton's Theorem

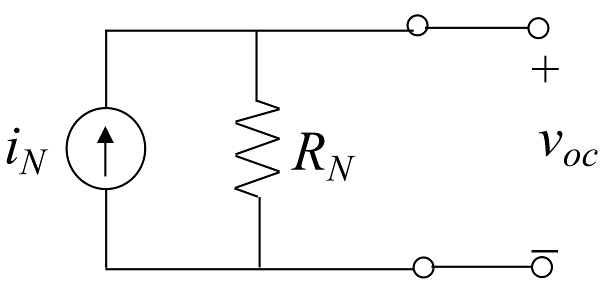
If the source cct in a two-terminal interface is linear, then the interface signals *v* and *i* do not change when the source cct is replaced by its Norton equivalent

Norton Equivalent Circuit

i_N is the short-cct current

R_T is evaluated from open-cct voltage

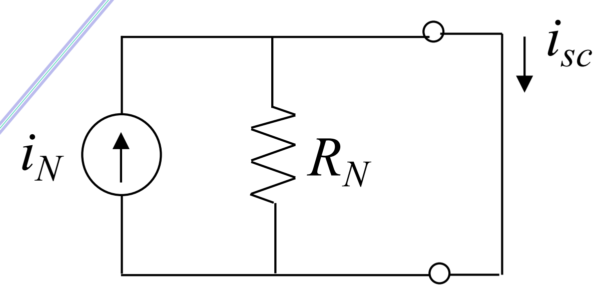




$$v_{oc} = i_N R_N = i_{sc} R_N$$

$$R_N = \frac{v_{oc}}{i_{sc}}$$

$$= R_T$$



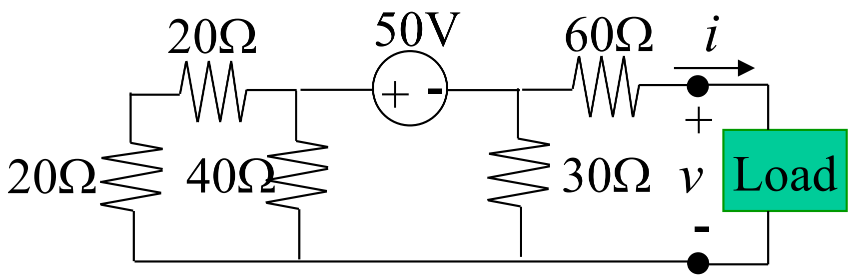
$$i_N = i_{sc}$$

$$= \frac{v_T}{R_N}$$

$$= \frac{v_T}{R_T}$$

Example 3-16 p.110 T&R, 5th ed

Find the Thévenin and Norton equivalent ccts of



Find the voltage, current and power if load is 50Ω

Answer: v_T=-30V; i_N=-417mA; R_N=R_T=72Ω

V=-12.3V; i=-246mA; p=3.03W