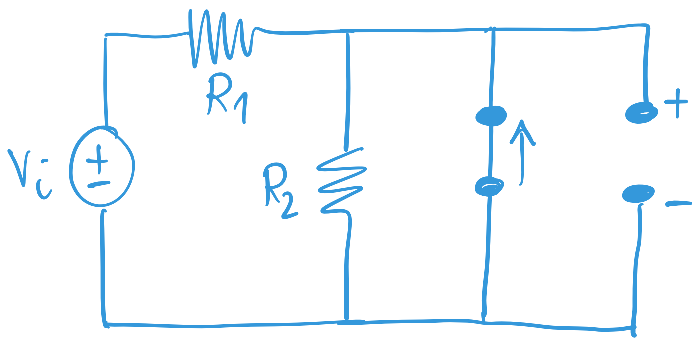
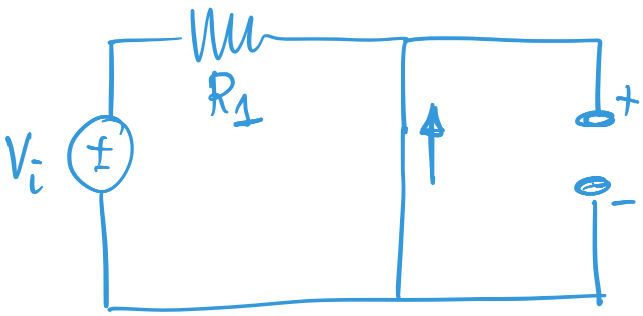
1._ Part I

Under DC excitations, we know the capacitor behaves as an open circuit and the inductor behaves as a short circuit. Therefore we have



[+1 extra point]

This circuit is equivalent to

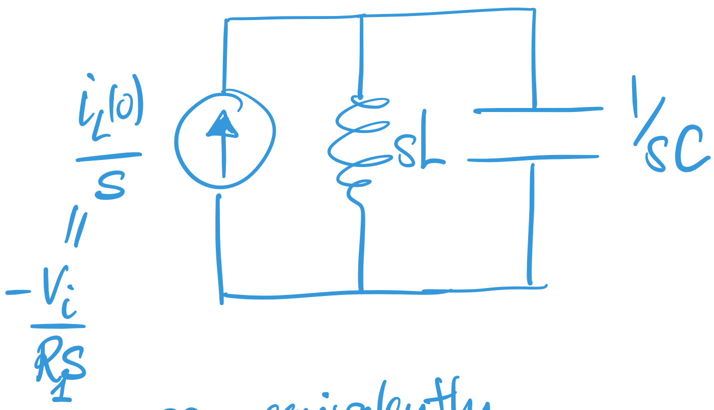


Therefore, we conclude that $i_L(0) = -\dfrac{V_i}{R_1}$ [+0.5 extra point]

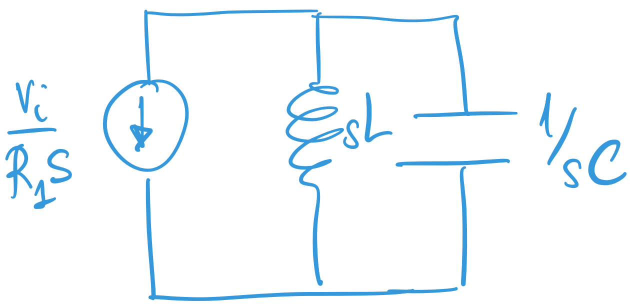
and $V_C(0) = 0$. [+0.5 extra point]

Part II

We redraw the circuit in the s-domain, using a current source to account for the initial condition of the inductor.



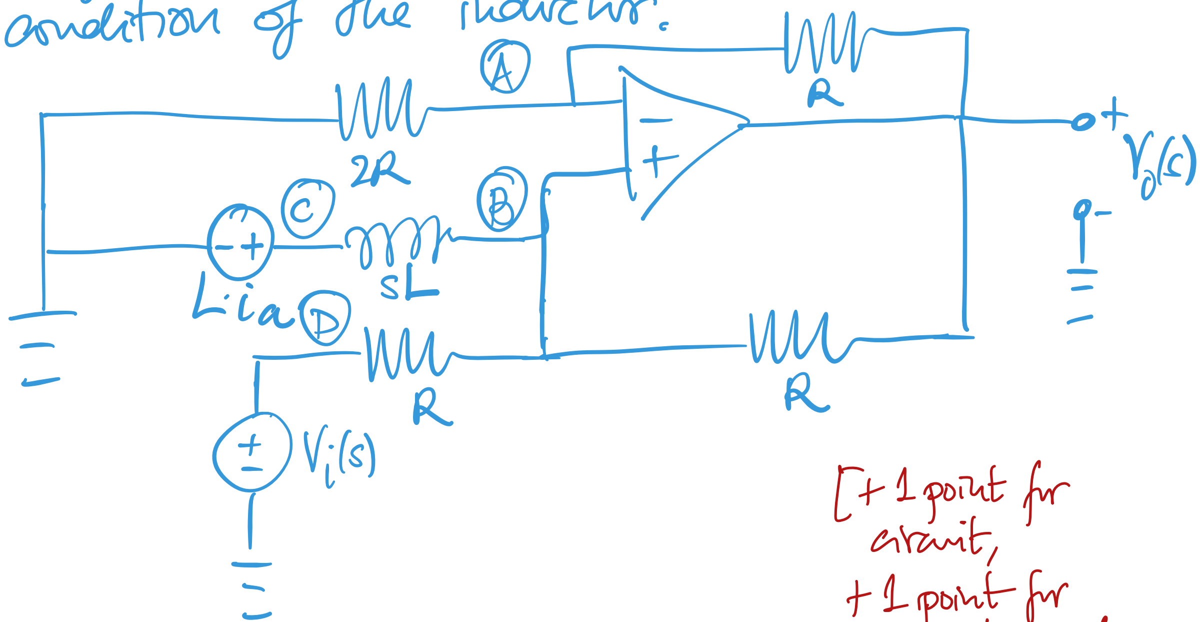
or equivalently



[+1 extra point]

2.– **Part I**

We transform the circuit into the S-domain, using a voltage source to represent the initial condition of the inductor.



[+1 point for circuit,
+1 point for correctly capturing initial condition of inductor]

Part II

We use the node labels provided in the plot above. Right away, we have that

$$V_C = Li_a \quad ; \quad V_D = V_i(s)$$

[+0.5 point] [+0.5 point]

Because of ideal op-amp conditions, we have

$$V_A = V_B$$ [+0,5 point]

KCL @ node A gives

$$\frac{1}{2R} V_A + \frac{1}{R}(V_A - V_o(s)) = 0$$ [+0,5 point]

KCL @ node B gives

[+0,5 point]

$$\frac{1}{sL}(V_B - V_C) + \frac{1}{R}(V_B - V_D) + \frac{1}{R}(V_B - V_o) = 0$$

This gives us a total of 5 eqs in 5 unknowns

V_A, V_B, V_C, V_D, V_o. [+0,5 point]

From the 4th equation, we obtain

$$V_o(s) = \frac{3}{2} V_A.$$

Substituting into the last equation, we get

$$\frac{1}{sL}(V_A - Li_a) + \frac{1}{R}(V_A - V_i(s)) + \frac{1}{R}\left(V_A - \frac{3}{2}V_A\right) = 0$$

Solving for V_A, we get $\qquad -\frac{1}{2}V_A$

$$\left(\frac{1}{sL}+\frac{1}{R}-\frac{1}{2R}\right)V_A=\frac{1}{sL}Li_a+\frac{1}{R}V_i(s)$$

$$\left(\frac{1}{sL}+\frac{1}{2R}\right)V_A=\frac{i_a}{s}+\frac{V_i(s)}{R}$$

$$\parallel$$

$$\frac{2R+sL}{2RLs}V_A \implies$$

$$V_A(s)=\frac{2RLs}{2R+sL}\left(\frac{i_a}{s}+\frac{V_i(s)}{R}\right)$$

Therefore

$$V_0(s)=\frac{3RL}{2R+sL}\left(i_a+\frac{s}{R}V_i(s)\right)=$$

$$=\frac{3L}{2R+sL}\left(Ri_a+sV_i(s)\right)$$

[+1 point]

Part III

Substituting the values provided, we get

$$V_0(s) = \frac{3(Ri_a + sV_i(s))}{\frac{2R}{L} + s} = \frac{3(10 \cdot 2 \cdot 10^{-1} + s \cdot 1/s^2)}{s + 2 \cdot \frac{10}{10^{-2}}} =$$

$$= \frac{3(2 + \frac{1}{s})}{s + 2000} = \frac{6}{s+2000} + \frac{3}{(s+2000)s}$$

$$= \frac{6}{s+2000} + \frac{A}{s+2000} + \frac{B}{s}$$

We use the residue method to find

$$A = \lim_{s \to -2000} \cancel{(s+2000)} \cdot \frac{3}{\cancel{(s+2000)}s} = -\frac{3}{2000}$$

$$B = \lim_{s \to 0} \cancel{s} \cdot \frac{3}{(s+2000)\cancel{s}} = \frac{3}{2000}$$

Therefore

$$V_o(s) = \frac{6}{s+2000} - \frac{3/2000}{s+2000} + \frac{3/2000}{s}$$

[+1 point]

Taking inverse Laplace transform, we obtain

$$v_o(t) = \left(\left(6 - \frac{3}{2000}\right) \cdot e^{-2000t} + \frac{3}{2000}\right) u(t)$$

$$= \left(\frac{11997}{2000} e^{-2000t} + \frac{3}{2000}\right) u(t)$$

[+1 point]

Part IV

The input is $v_i(t) = t\,u(t)$, which has two poles at zero. Therefore, the forced response is

$$v_{fr}(t) = \frac{3}{2000} u(t)$$

[+0.5 point]

and the natural response is

$$v_{nr}(t) = \frac{11997}{2000} e^{-2000t} u(t)$$

[+0.5 point]

(as expected, the natural response almost immediately becomes zero)

The zero-state response is obtained by zeroing the initial condition i_a of the inductor

$$V_{ozs}(s) = -\frac{3}{2000}\,\frac{1}{s+2000} + \frac{3}{2000}\cdot\frac{1}{s}$$

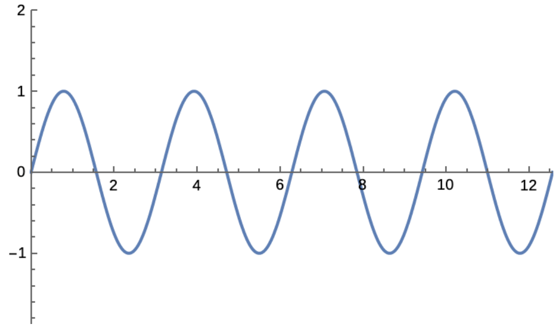
$$V_{ozs}(t) = \left(-\frac{3}{2000}\,e^{-2000t} + \frac{3}{2000}\right)u(t)$$ [+0.5 point]

The zero-input response is obtained by zeroing the input $V_i(t)$,

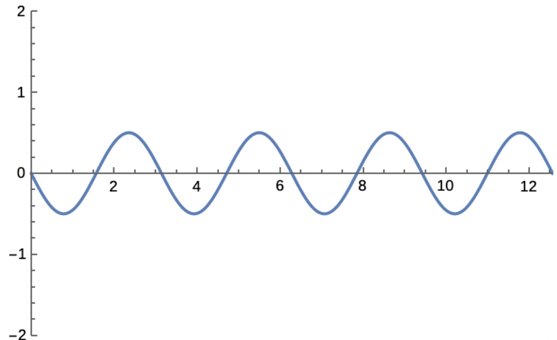
$$V_{ozi}(s) = 6\cdot\frac{1}{s+2000}$$

$$V_{ozi}(t) = 6\cdot e^{-2000t}\,u(t)$$ [+0.5 point]

3. Part I For circuit 1, we have



← input

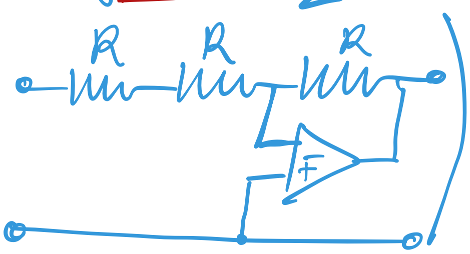


← steady-state output

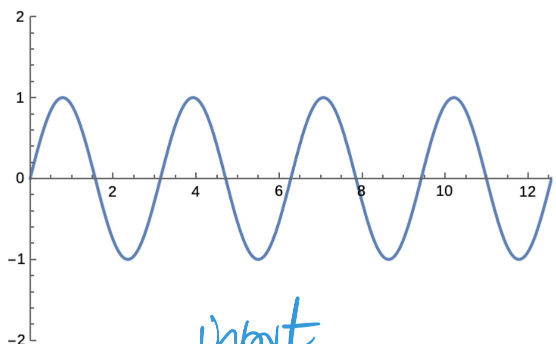
We see that the circuit is inverting the input, and scaling it down by $1/2$. So this must be an inverting op-amp, with gain $-1/2$.

[+1 point]

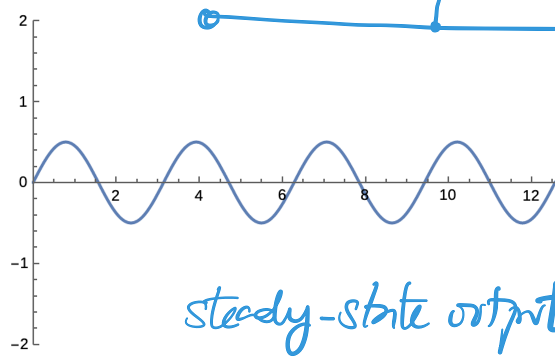
(e.g.,



For circuit 2, we have

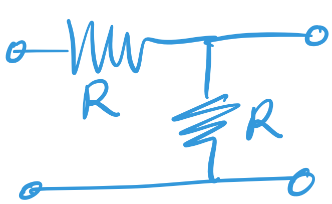


input



steady-state output

We see that the circuit is scaling down the input by a factor of $1/2$, respecting the polarity. This must be a voltage divider, with gain $1/2$.

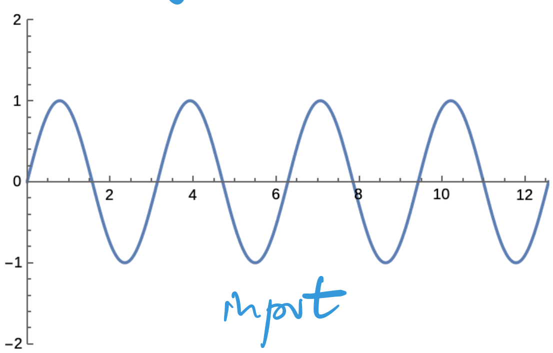
(e.g., 

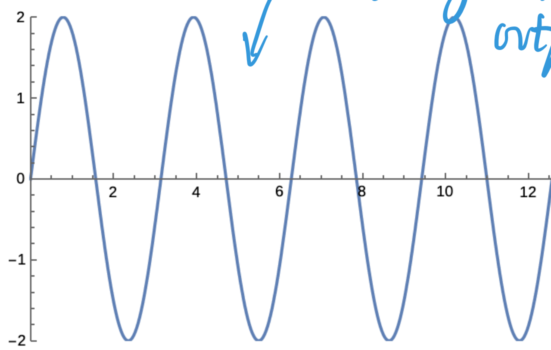
)

[+1 point]

Finally, for circuit 3 we have

steady-state output

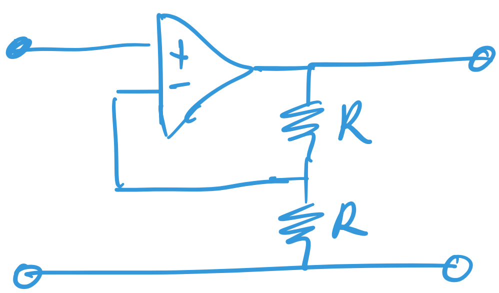




input

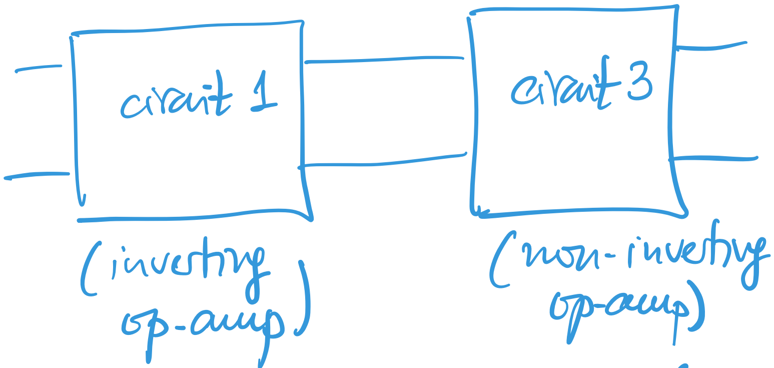
The circuit respects the polarity, and amplifies the input by a factor of 2, so this must be a non-inverting op-amp, with gain 2.

[+1 point]

(e.g., 

)

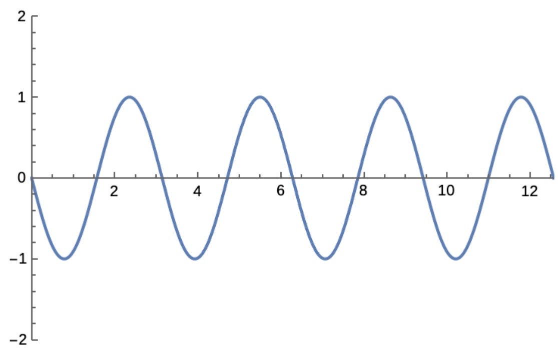
Part II



There is no loading (because of both the 0-output impedance of circuit 1 and the ∞-input impedance of circuit 3), so

$$T(s) = T_1(s) \cdot T_3(s) = -\frac{1}{2} \cdot 2 = -1$$ [+0.5 point]

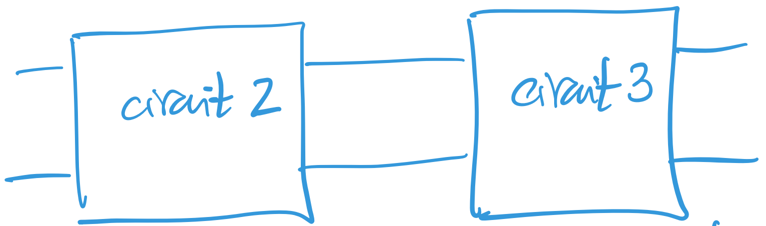
Therefore, the steady-state response of the connection in series is



[+1 point]

The order in which the circuits are connected does not matter (there is no loading in either case). [+0.5 point]

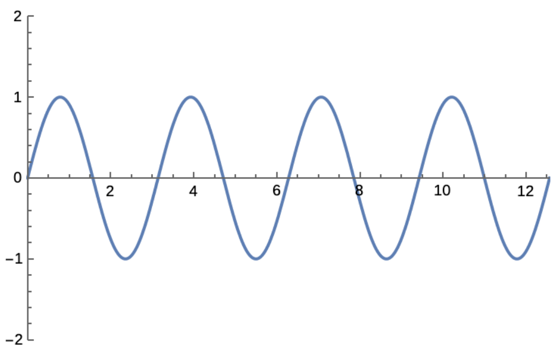
Part III



There is no loading (because of the ∞-input impedance of circuit 3), so

$$T(s) = T_2(s) \cdot T_3(s) = \frac{1}{2} \cdot 2 = 1$$ [+0.5 point]

Therefore, the steady-state response of the connection in series is

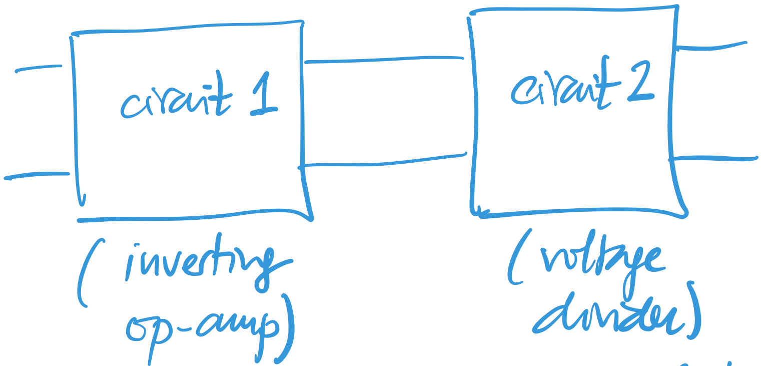


[+1 point]

The order in which the circuits are connected does not matter (there is no loading if we connect −[C3]−[C2]−, because of the 0-output impedance of circuit 3)

[+0.5 point]

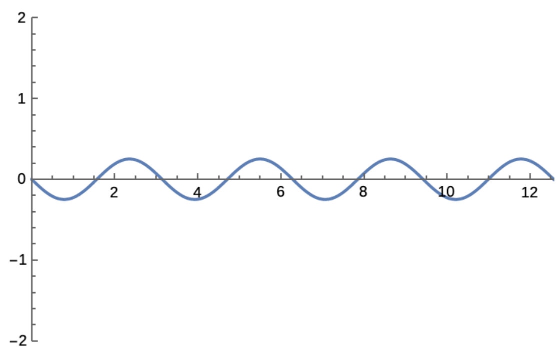
Part IV



There is no loading (because of the 0-output impedance of circuit 1), so

$$T(s) = T_1(s) \cdot T_2(s) = -\frac{1}{2} \cdot \frac{1}{2} = -\frac{1}{4}$$ [+0.5 point]

Therefore, the steady-state response of the connection in series is

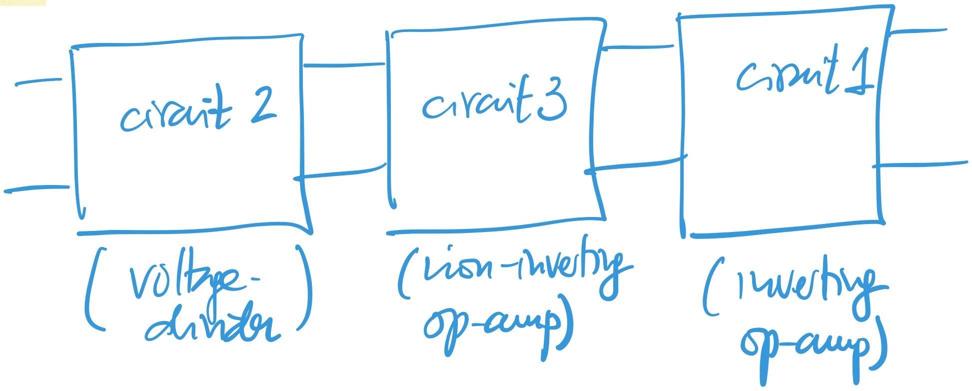


[+1 point]

The order in which the circuits are connected does matter – if we change the order, there would be loading.

[+0.5 point]

Part V



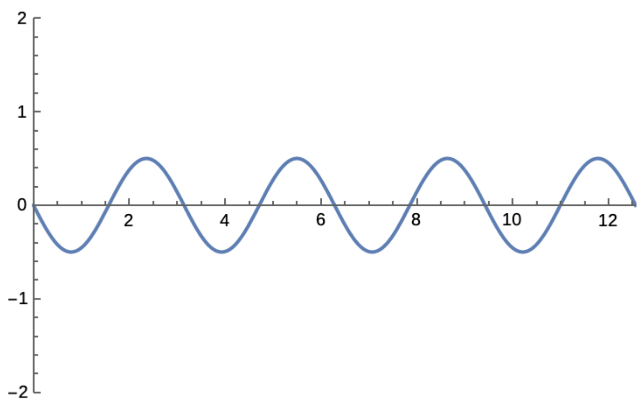
The presence of the non-inverting op-amp in between the divider and the inverting op-amp makes sure that there is no loading. Therefore,

$$T(s) = T_2(s) \cdot T_3(s) \cdot T_1(s) =$$

$$= \frac{1}{2} \cdot 2 \cdot \left(-\frac{1}{2}\right) = -\frac{1}{2}$$

[+1 extra point]

Therefore, the steady-state response of the connection in series is



[+1 extra point]

4. Part I

To compute the gain and phase functions, we evaluate the transfer function at $s = j\omega$,

$$T(j\omega) = -\frac{-10\omega^2 + 2000j\omega + 10^6}{-\omega^2 + 1100j\omega + 10^5} =$$

$$= -\frac{(10^6 - 10\omega^2) + (2000\omega)j}{(10^5 - \omega^2) + (1100\omega)j}$$

Therefore

$$|T(j\omega)| = \frac{\sqrt{(10^6 - 10\omega^2)^2 + 2000^2\omega^2}}{\sqrt{(10^5 - \omega^2)^2 + 1100^2\omega^2}}$$ [+1 point]

$$\angle T(j\omega) = \underbrace{\arctan \frac{2000\omega}{10^6 - 10\omega^2}} - \arctan \frac{1100\omega}{10^5 - \omega^2}$$

(when computing angles, remember this corresponds to complex number $-(10^6 - 10\omega^2) - 2000\omega j$) [+1 point]

Part II

$$|T(j0)| = \frac{\sqrt{(10^6)^2}}{\sqrt{(10^5)^2}} = \frac{10^6}{10^5} = 10$$ [+0.5 point]

$$|T(j\infty)| = \lim_{\omega \to \infty} |T(j\omega)| = \lim_{\omega \to \infty} \frac{\sqrt{100\omega^4}}{\sqrt{\omega^4}} = 10$$ [+0.5 point]

$\angle T(j0) = \pi - 0 = \pi$ [+0.5 point]

$\angle T(j\infty) = 2\pi - \pi = \pi$ [+0.5 point]

To compute the cutoff frequencies, we have to compute the maximum value of the gain function.

$$|T(j\omega)| = \sqrt{\frac{(10^6 - 10\omega^2)^2 + 2000^2\omega^2}{(10^5 - \omega^2)^2 + 1100^2\omega^2}} =$$

$$= \sqrt{\frac{10^{12} + 100\omega^4 - 2\cdot 10^7\omega^2 + 4\cdot 10^6\omega^2}{10^{10} + \omega^4 - 2\cdot 10^5\omega^2 + 12.1\cdot 10^5\omega^2}}$$

$$= \sqrt{\frac{10^{12} + 100\omega^4 - 16\cdot 10^6\omega^2}{10^{10} + \omega^4 + 10.1\cdot 10^5\omega^2}}$$

Thankfully, they provide us with the maximum value, $T_{max} = 10$

Therefore, the cutoff frequencies are found by solving

$$\boxed{|T(j\omega_c)| = \frac{T_{max}}{\sqrt{2}}} = \frac{10}{\sqrt{2}}$$

$$\parallel$$

$$\sqrt{\frac{10^{12} + 100\omega_c^4 - 16\cdot 10^6 \omega_c^2}{10^{10} + \omega_c^4 + 10.1\cdot 10^5 \omega_c^2}}$$

[+1 point]

$$\frac{10^{12} + 100\omega_c^4 - 16\cdot 10^6 \omega_c^2}{10^{10} + \omega_c^4 + 10.1\cdot 10^5 \omega_c^2} = \frac{100}{2} = 50$$

$$10^{12} + 100\omega_c^4 - 16\cdot 10^6 \omega_c^2 = 5\cdot 10^{11} + 50\omega_c^4 + 505\cdot 10^5 \omega_c^2$$

$$50\omega_c^4 - 665\cdot 10^5 \omega_c^2 + 5\cdot 10^{11} = 0$$

$$\omega_c^4 - 133\cdot 10^4 \omega_c^2 + 10^{10} = 0 \qquad \omega_c^2 = z$$

$$z^2 - 133\cdot 10^4 z + 10^{10} = 0$$

$$z = \frac{133\cdot 10^4 \pm \sqrt{133^2\cdot 10^8 - 4\cdot 10^{10}}}{2} =$$

$$= \frac{133 \cdot 10^4 \pm 10^4 \cdot \sqrt{17289}}{2} = \begin{cases} 132.2438 \cdot 10^4 \\ 0.7561 \cdot 10^4 \end{cases}$$

Therefore

$$w_c = \sqrt{z} \simeq \begin{cases} 1149.97 \text{ rad/s} & \text{[+0.5 point]} \\ 86.95 \text{ rad/s} & \text{[+0.5 point]} \end{cases}$$

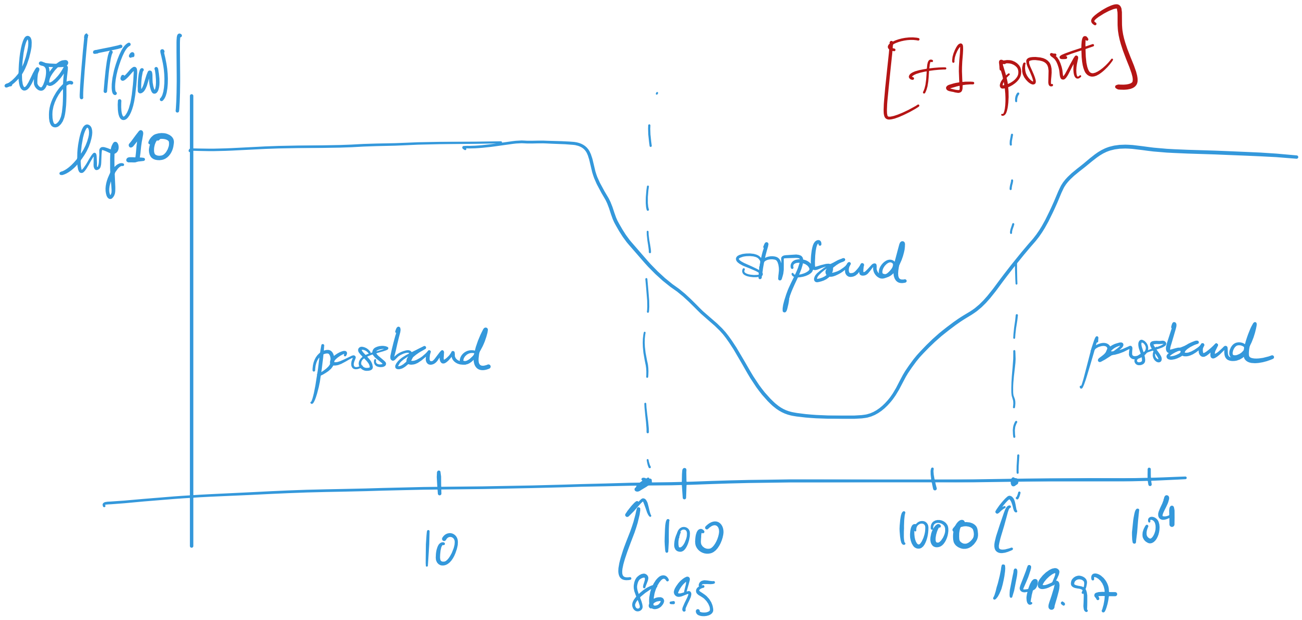
Part III

For $w \ll 1$, $|T(jw)| \approx \sqrt{\dfrac{10^{12}}{10^{10}}} = 10$

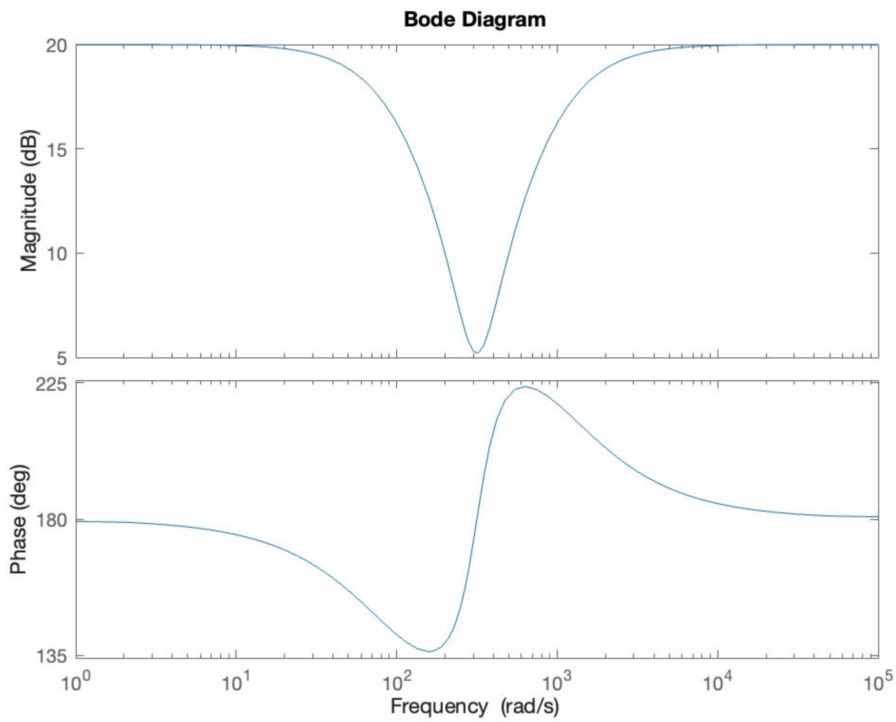
For $w \gg 1$, $|T(jw)| \simeq \sqrt{\dfrac{100 w^4}{w^4}} = 10$

Sketchs can be done in linear or logarithmic scale.

[+1 point]



This is a bandstop filter. [+1 point]



Part IV

The three inputs only differ in their frequencies.

We have

$|T(j250)| \simeq 2.25$ $\qquad$ $\angle T(j250) \simeq 2.63$ rad

$|T(j300)| \simeq 1.84$ $\qquad$ $\angle T(j300) \simeq 3.00$ rad

$|T(j350)| \simeq 1.90$ $\qquad$ $\angle T(j350) \simeq 3.39$ rad

[+1 point]

Therefore, the steady-state responses are

$$V_{01}^{ss}(t) = A \cdot 2.25 \cos(250t + \Phi + 2.63)$$

$$V_{02}^{ss}(t) = A \cdot 1.84 \cos(300t + \Phi + 3.00)$$

$$V_{03}^{ss}(t) = A \cdot 1.90 \cos(350t + \Phi + 3.39)$$ [+1 point]

Yes, the transfer function reasonably accomplished the engineer's goal. This is because all signals with frequencies in the range 250-350 rad/s (the ones used by the transmitter) fall into the stopband, with gains of around 2. Instead, the passband gain is 10. This means that the interference was greatly reduced for the radio receiver.

[+1 point]

5.. Part I

We find the poles of the transfer function

$$s^2 + 1100s + 10^5 = 0$$

$$s = \frac{-1100 \pm \sqrt{1100^2 - 4 \cdot 10^5}}{2} = \frac{-1100 \pm 900}{2} = \begin{cases} -100 \\ -1000 \end{cases}$$

[+1 point]

Therefore, we can write

$$T(s) = \frac{-k_1 s}{\underbrace{s + 1000}_{\alpha_1}} + \frac{-k_2}{\underbrace{s + 100}_{\alpha_2}} =$$

$$= \frac{-k_1 s^2 - k_1 100s - k_2 s - k_2 1000}{s^2 + 1100s + 10^5}$$

Since denominators are the same, numerators have to be too. Therefore, equating coefficients,

$$k_1 = 10$$

$$100k_1 + k_2 = 2000 \Rightarrow k_2 = 1000$$

$$k_2 1000 = 10^6 \checkmark$$

[+1 point]

So we finally write

$$T(s) = -\frac{10s^2 + 2000s + 10^6}{s^2 + 1100s + 10^5} = -\frac{10s}{s+1000} - \frac{1000}{s+100}$$

Part II

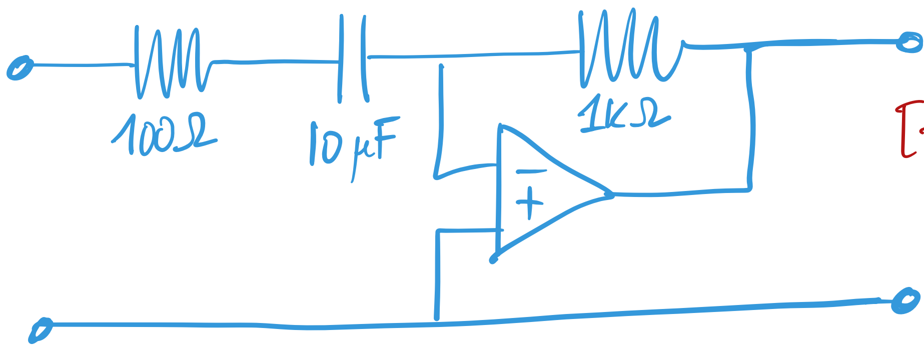
$$T_1(s) = \frac{-10s}{s+1000} = \frac{-10}{1+\frac{10^3}{s}} = \frac{-10^3}{10^2 + \frac{1}{s10^{-5}}}$$

[+1 point]

$$T_2(s) = -\frac{1000}{s+100} = -\frac{10^3/s}{1+\frac{10^2}{s}} = -\frac{1/s10^{-6}}{10^3+\frac{1}{s10^{-5}}}$$

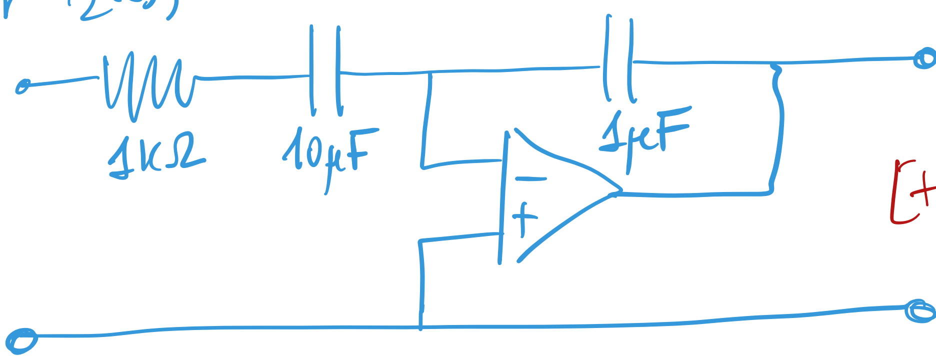
[+1 point]

We design circuits using inverting op amps.
For $T_1(s)$, we use



[+1 point]

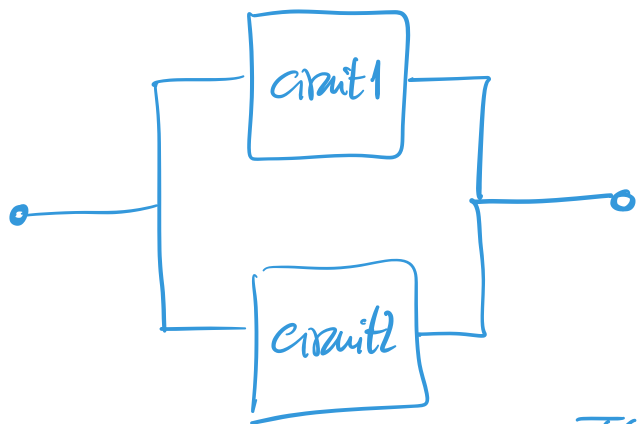
For $T_2(s)$, we use



[+1 point]

Part III

Since $T(s)$ is the sum of $T_1(s)$ and $T_2(s)$, we combine the two circuits in Part II in parallel to design [+1 point]

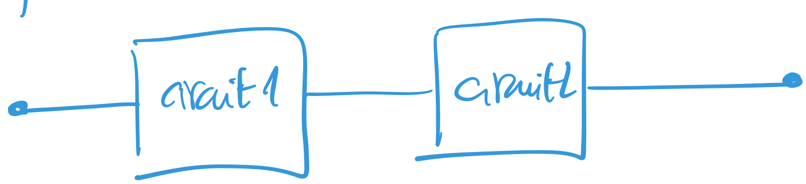


[+1 point]

With this design, we have $T(s) = T_1(s) + T_2(s)$.

Part IV

If we connect the circuits in series,

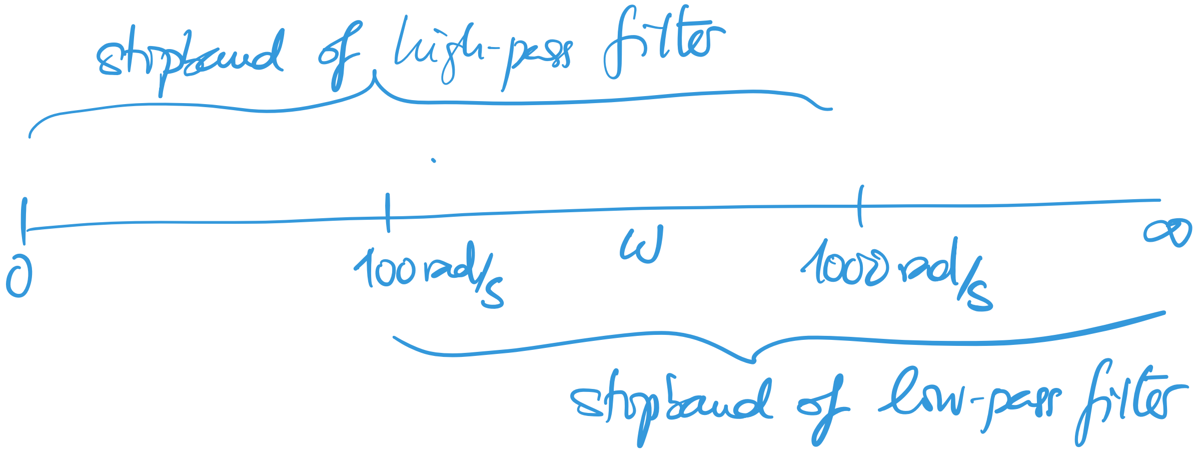


Then the resulting transfer function is

$$T_{series}(s) = T_1(s) \cdot T_2(s)$$

(Note that there is <u>no loading</u> because of the zero-output impedance of the op-amp). [+1 point]

This connection in series results in a filter with <u>no passband</u>. This is because $T_1(s)$ is a high-pass filter w/ cut-off freq $\omega_1 = \alpha_1 = 1000$ rad/s and $T_2(s)$ is a low-pass filter w/ cut-off freq $\omega_2 = \alpha_2 = 100$ rad/s. If we combine them in parallel (as in Part III), they give rise to the bandstop filter of Q4. But if we combine them in series, as in this Part IV, then no frequency passes through.



So there is <u>no passband</u>. [+1 point]