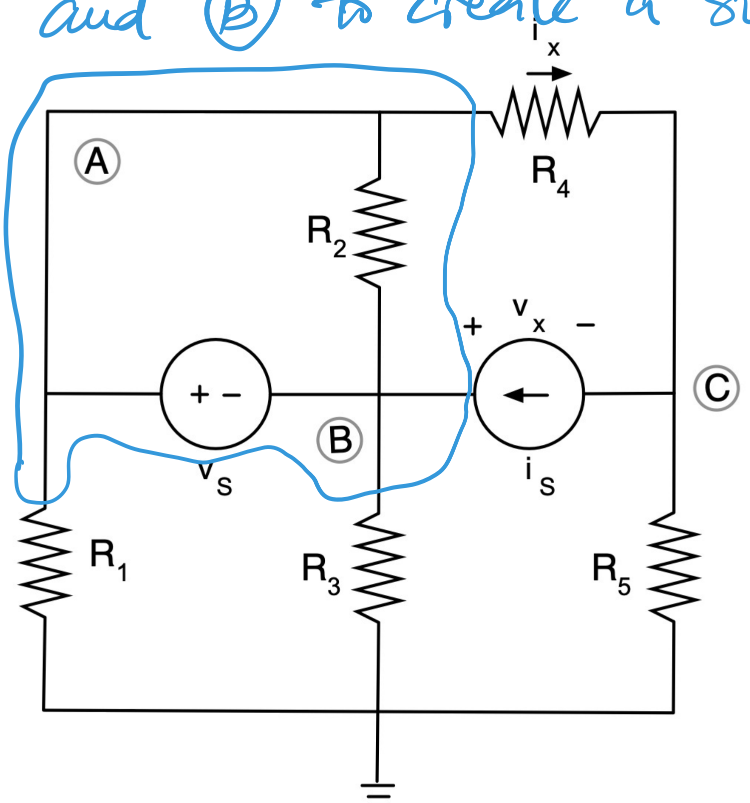
1.– Part I

To use node-voltage analysis, we must take care of the presence of the voltage source using one of the three methods discussed in class:

1) source transformation
2) grounding a node conveniently
3) supernode

We cannot use 1) because the voltage source is not in series with a resistor (even if it was, the statement of the question explicitly rules out modifying the circuit, which also discards source transformation). 2) cannot be used either, because the ground node (which has already been chosen) is not placed conveniently. So we are left with method 3), where we combine nodes A and B to create a supernode.



[+2 points]

The equation for the supernode is

$$V_A - V_B = V_S$$ [+1 point]

Next, we write KCL for the supernode,

$$G_1(V_A) + G_3 V_B + G_4(V_A - V_C) = i_S$$ [+1 point]

(Here, we have used the shorthand notation $G_i = \frac{1}{R_i}$).

Next, we write KCL for node Ⓒ,

$$G_4(V_C - V_A) + G_5 V_C + i_S = 0$$ [+1 point]

In matrix form, we have

$$\begin{pmatrix} 1 & -1 & 0 \\ G_1+G_4 & G_3 & -G_4 \\ -G_4 & 0 & G_4+G_5 \end{pmatrix} \begin{pmatrix} V_A \\ V_B \\ V_C \end{pmatrix} = \begin{pmatrix} V_S \\ i_S \\ -i_S \end{pmatrix}$$ [+1 point]

This gives 3 equations in 3 unknowns.

Part II

In terms of the node voltages, we have

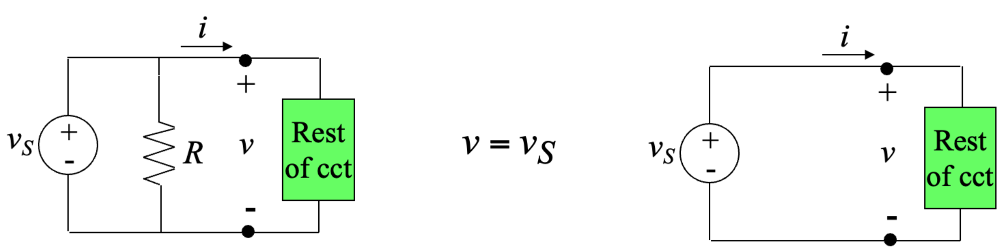
$$v_x = V_B - V_C \qquad \text{[+1 point]}$$

$$i_x = G_4(V_A - V_C) \qquad \text{[+1 point]}$$

Part III

Changing the value of R_2 would <u>not affect</u> the value of v_x and i_x. This can be justified in various ways. For instance, looking at the equations we wrote in Part I, R_2 does not appear in any. That means that its value does not affect the value of the node voltages, hence it does not affect the value of v_x and i_x. Another way to justify it is to realize that R_2 is in parallel with the voltage source.

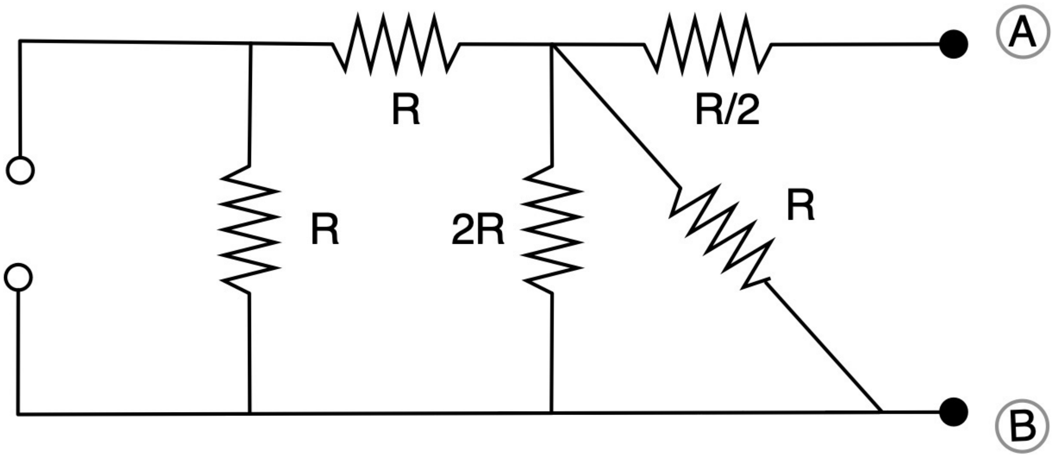
[+2 points] (either justification is valid)



This means that, from the point of view of the rest of the circuit, nothing changes if we eliminate it. So its value does not affect v_x & i_x.

2.- Part I

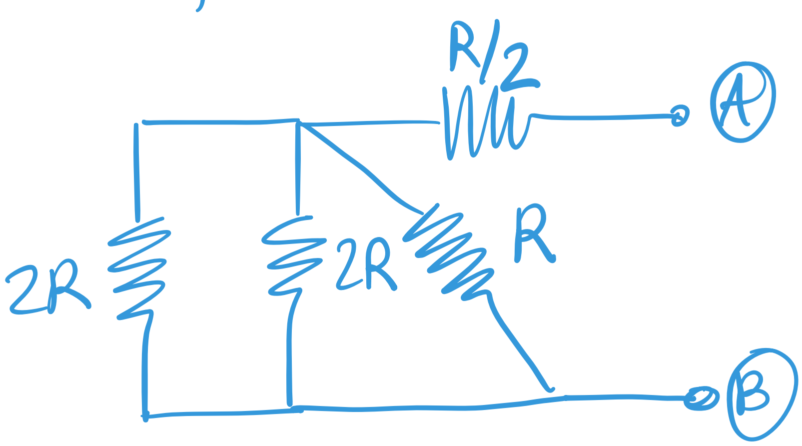
We turn off the source in the circuit and obtain the circuit below



where the current source gets replaced by an open circuit. [+1 point]

Next, we use association of resistors to find the equivalent resistance.

The two R-resistors on the left are in series, so we combine them

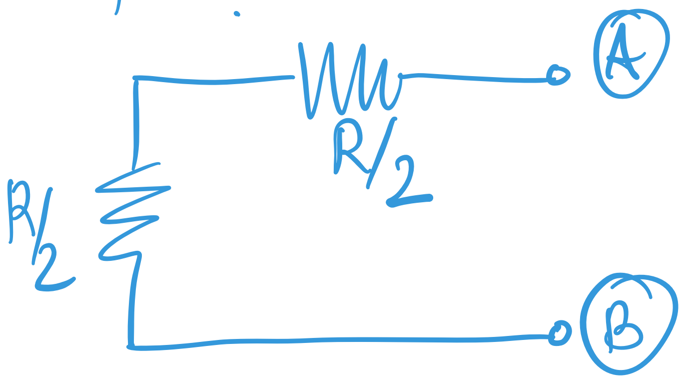


[+1 point]

The resulting resistor is in parallel with the vertical 2R-resistor, which is also in parallel with the R-resistor. So we compute

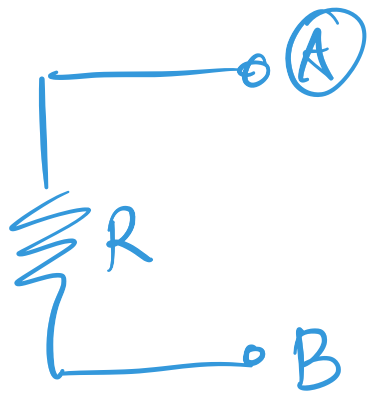
$$2R \| 2R \| R = \frac{R}{2}$$

and plot



[+1 point]

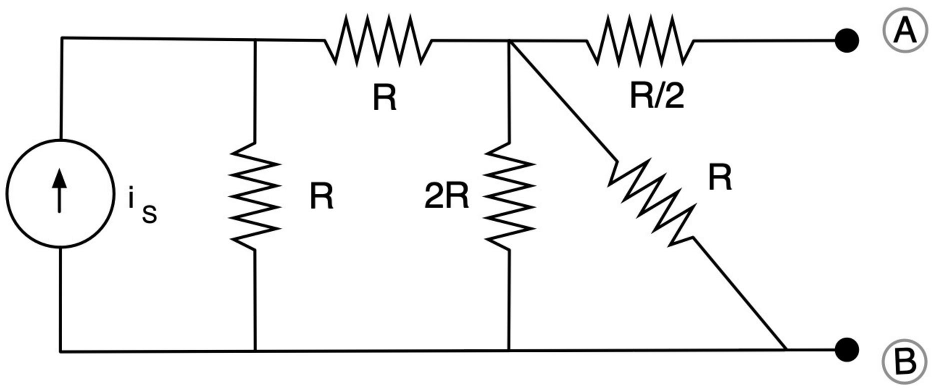
Finally, the two $\frac{R}{2}$-resistors are in series, so we conclude



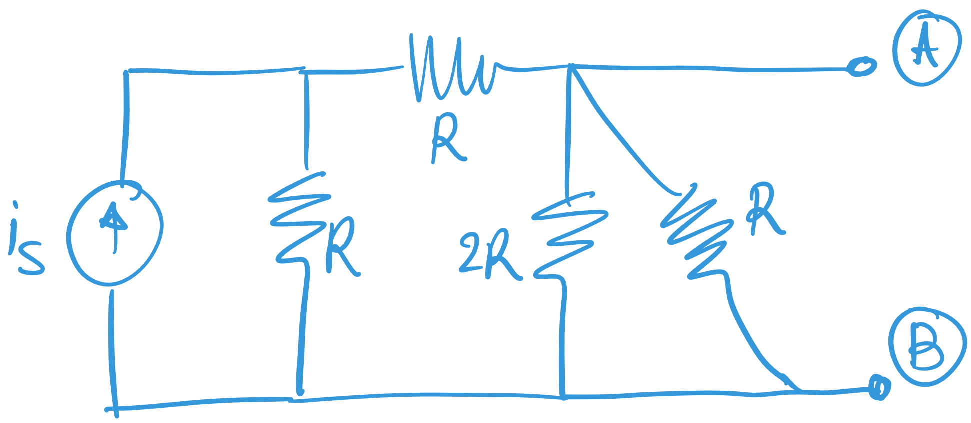
$$R_{AB} = R$$

[+1 point]

Part II



We are told to compute the open-circuit voltage as seen from terminals Ⓐ Ⓑ. Since there is no current flowing through the $R/2$ resistor, we can simply consider the following

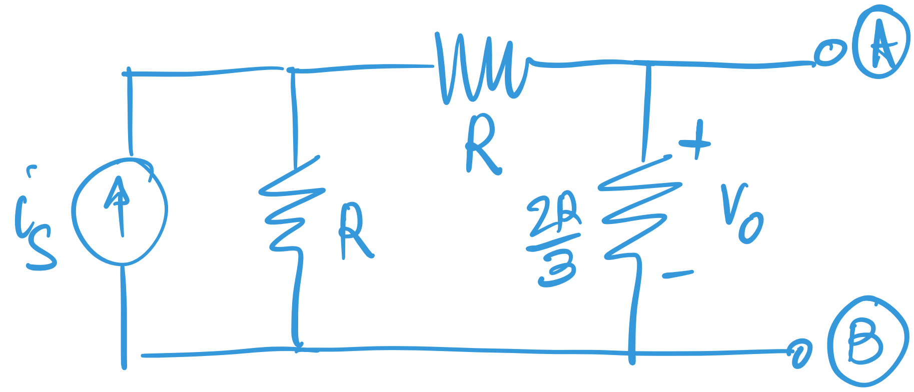


[+1 point]

So the voltage drop we are to compute is the one seen by the R-resistor on the right (which is the same as the one seen by the 2R-resistor, since they are in parallel).

We combine the resistors in parallel & obtain

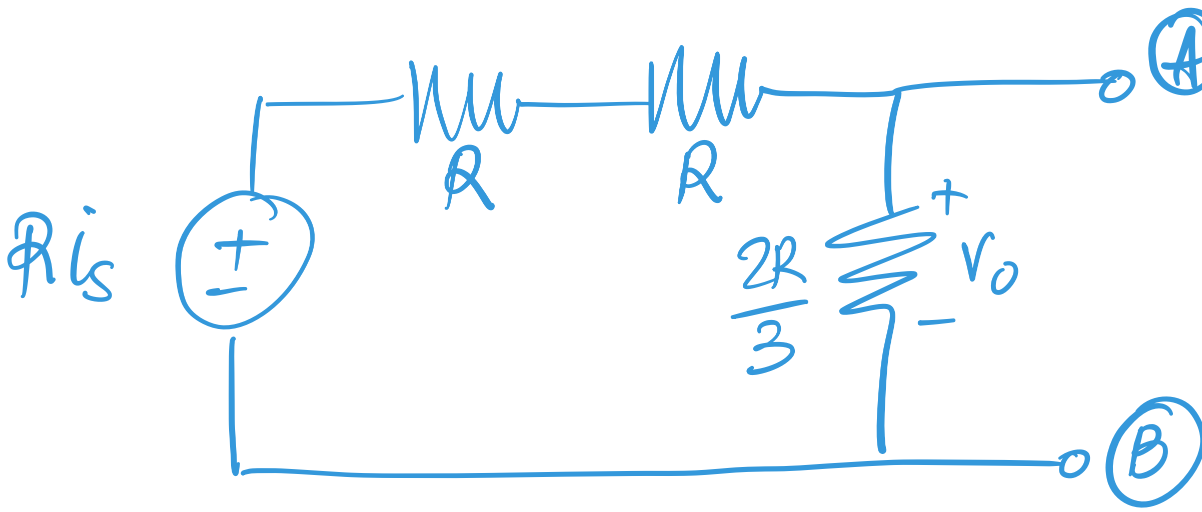
$$2R // R = \frac{2R^2}{3R} = \frac{2R}{3}$$



[+1 point]

There are several ways to compute the voltage V_o. For instance, we could use current division to find out the current going through the resistors R & $\frac{2R}{3}$ in series, then use that information to find V_o.

Instead, we opt for a different route. We use source transformation to draw



[+1 point]

Now we can use voltage division to obtain

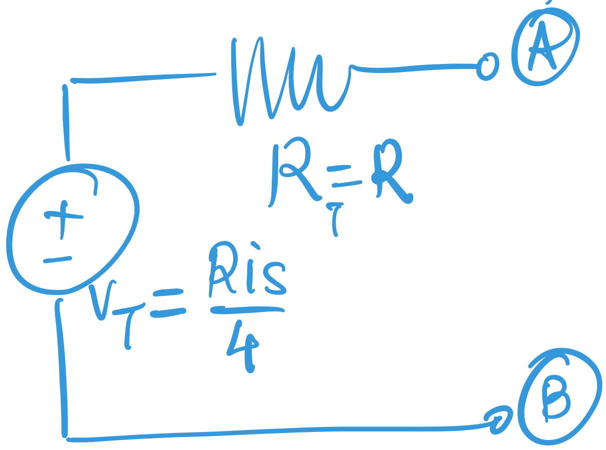
$$V_{AB} = V_o = \frac{2R/3}{2R/3 + R + R} Ri_s = \frac{2R/3}{8R/3} Ri_s = \frac{Ri_s}{4}$$

[+1 point]

Part III

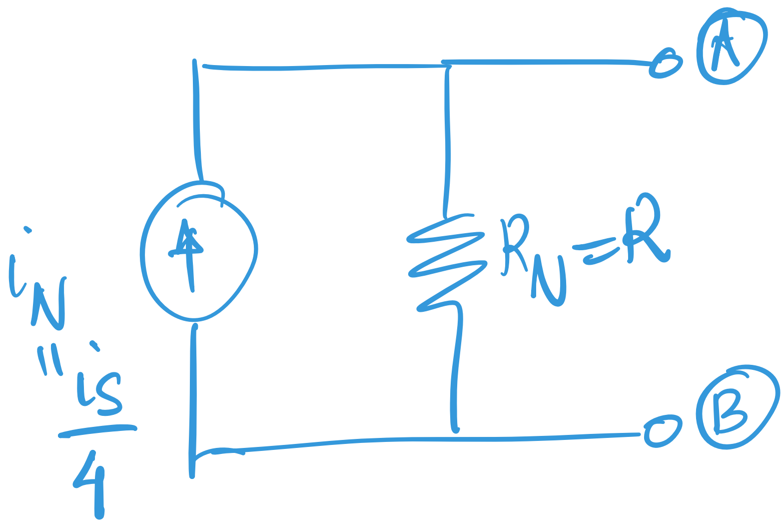
We already have all the information we need from our answers to Parts I & II.

The Thévenin equivalent circuit is



[+1 point]

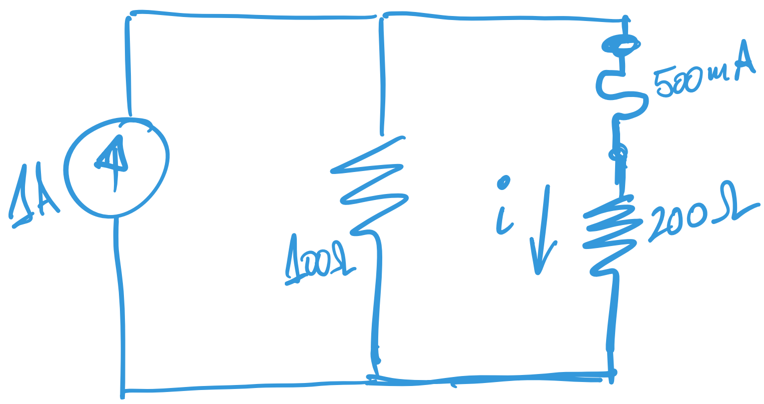
The Norton equivalent circuit is



[+1 point]

Part IV

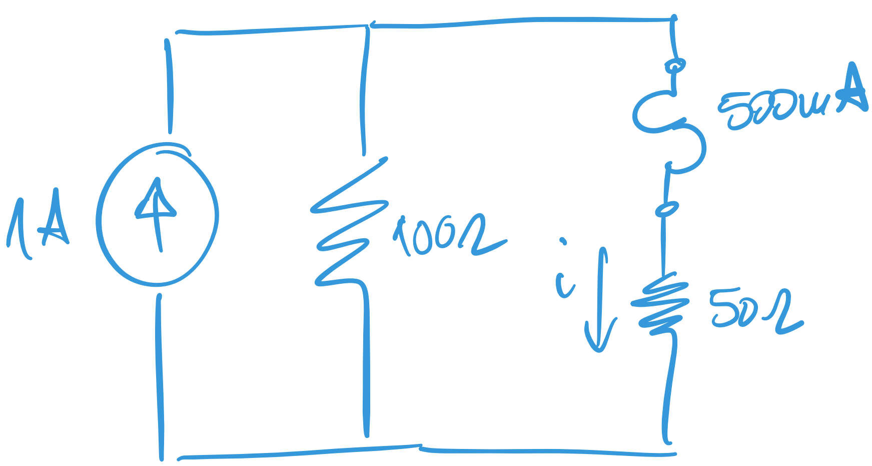
Connecting a fuse and a resistor in series results in



Using current division, we have $i = \frac{1/200}{1/200 + \frac{1}{100}} \cdot 1 = 0.33\,A = 330\,mA$

Since $330\,mA < 500\,mA$, the fuse does not blow. [+1 extra point]

However, if the value of the resistor is 50Ω, then for



We have

$$i = \frac{1/50}{\frac{1}{50} + \frac{1}{100}} \, 1 = 0.66\,A = 666\,mA$$

Since $666\,mA > 500\,mA$, the fuse will blow. [+1 extra point]