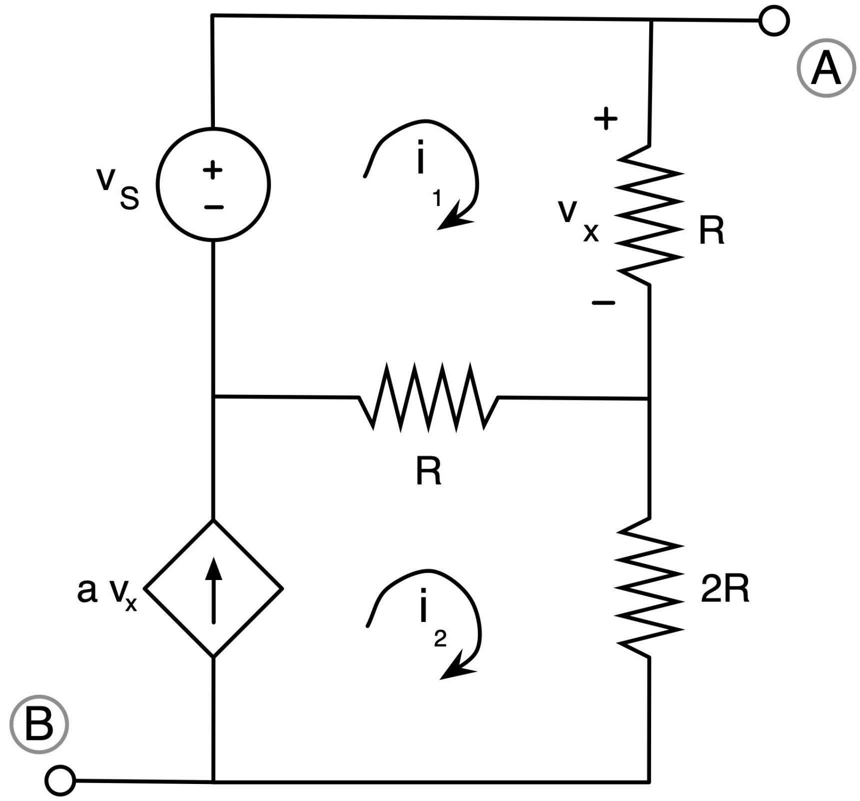
1.– Part I



Looking at the circuit, we observe the presence of 1 current source, which is a problem we need to deal with to use mesh current analysis.

The current source belongs to one mesh, so we just use $i_2 = a V_x$ [method 2] [+1 point]

KVL for the top mesh reads

$$R i_1 + R(i_1 - i_2) - V_S = 0$$ [+1 point]

We also need to account for the presence of the dependent source. Looking at the circuit, we see that

$$V_x = R i_1$$ [+1 point]

This discussion leads to a total of 3 eqs in 3 unknowns i_1, i_2, V_x. We now solve them to find the unknowns.

$$i_2 = aV_x = aRi_1$$

Then

$$2Ri_1 - aR^2 i_1 = V_S \Rightarrow \quad R(2-aR)i_1 = V_S$$

$$i_1 = \frac{V_S}{R(2-aR)}$$

Therefore, the open circuit voltage as seen from terminals A and B is

$$V_{OC} = V_{AB} = Ri_1 + 2Ri_2 = Ri_1 + 2R^2 a i_1 =$$

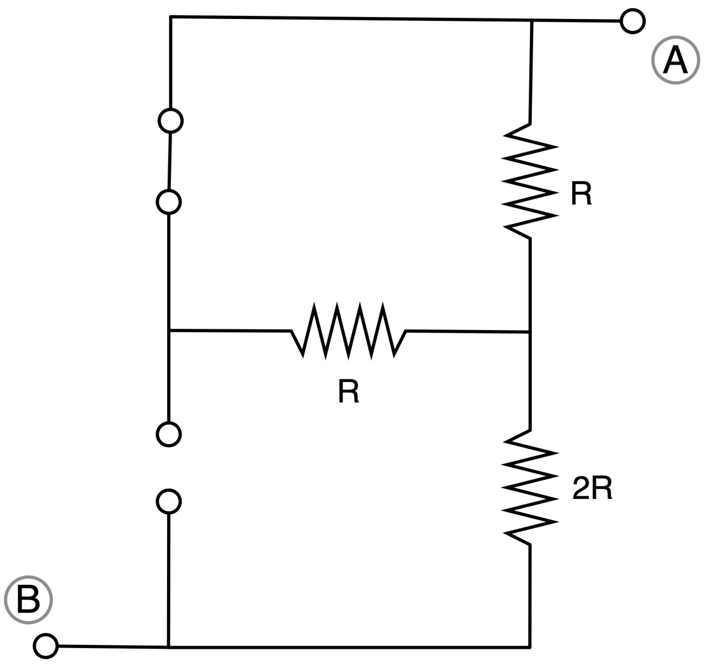
$$= \frac{\cancel{R}(1+2aR)}{\cancel{R}(2-aR)} V_S \qquad [+1 \text{ point}]$$

Part II

When we turn off the independent source, the dependent one also gets turned off. This can be seen in the expression for the mesh

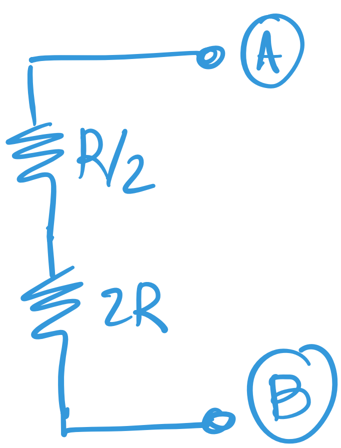
[+0.5 point]

currents in Part I: if $V_s = 0$, then both $i_1 = 0 = i_2$, and hence $V_x = 0$. With the sources off, the circuit looks like



[+0.5 point]

Note that the two R resistors are in parallel, so we draw

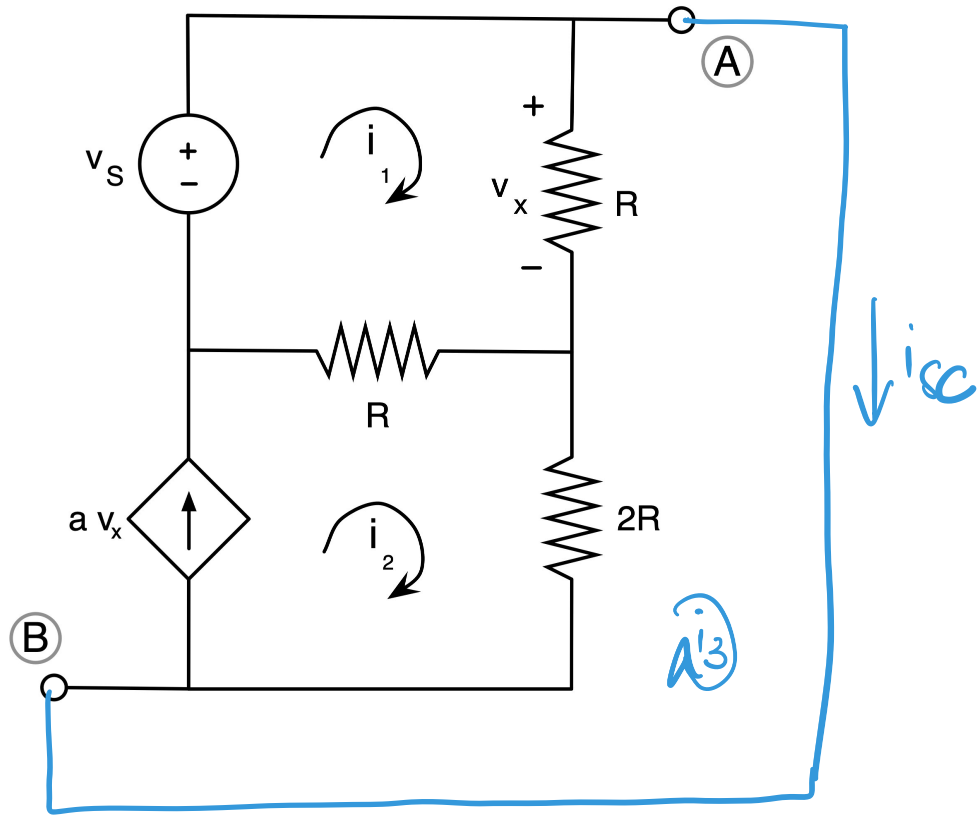


Combining the resistors in series, we obtain

$$R_{EQ} = 2R + R/2 = \frac{5R}{2}$$ [+1 point]

Part III

We connect terminals A and B and find the short-circuit current as



The short-circuit current is $i_3 = i_{SC}$. We use mesh-current analysis to find i_1, i_2, i_3 (and v_x).

We deal with the current source by writing

$$i_2 = a v_x \quad (\text{method } 2)$$ [+0.5 point]

We write KVL for meshes 1 & 3.

KVL for mesh 1 : $R(i_1 - i_3) + R(i_1 - i_2) - V_S = 0$

[+0.5 point]

KVL for mesh 3: $2R(i_3-i_2)+R(i_3-i_1)=0$

[+0.5 point]

We account for the presence of the dependent source with

$$V_x = R(i_1-i_3)$$ [+0.5 point]

So we have 4 eqs in 4 unknowns (i_1, i_2, i_3, V_x). Solving,

$$3i_3 - 2i_2 - i_1 = 0 \quad (\text{from } 3^{rd} \text{ eq})$$

So $i_1 = 3i_3 - 2i_2$

Substituting in 2nd eq,

$$2R(i_3-i_2)+3R(i_3-i_2) = V_S \Rightarrow 5R(i_3-i_2)=V_S$$

And

$$i_2 = aV_x = aR(i_1-i_3) = 2aR(i_3-i_2)$$

So $i_2 = \dfrac{2a\cancel{R}V_S}{5\cancel{R}} = \dfrac{2a}{5}V_S$

Therefore, $i_{sc} = i_3 = i_2 + \dfrac{V_S}{5R} = \dfrac{1+2aR}{5R}V_S$

[+1 point]

Part IV

With the answers to Parts I & II, we have

[+0.25 point]

$$V_T = V_{AB} = \frac{1+2aR}{2-aR}V_S$$

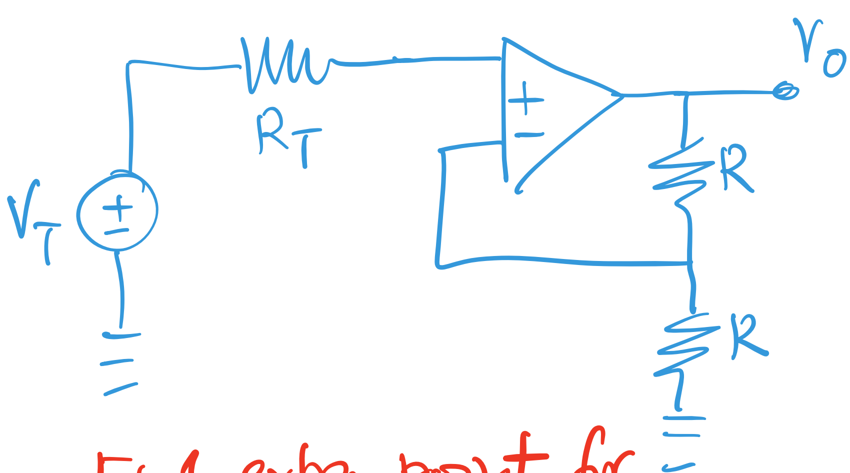
$$R_T = R_N = \frac{V_T}{i_{sc}} = \frac{\cancel{1+2aR}}{2-aR}\cancel{V_S} \cdot \frac{5R}{\cancel{1+2aR}} \cdot \frac{1}{\cancel{V_S}} = \frac{5R}{2-aR}$$ [+0.5 point]

$$i_N = i_{SC} = \frac{1+2aR}{5R} V_S$$ [+0.25 point]

The reason why $R_T = R_N$ is not equal to R_{EQ} computed in Part II is because, when we turn off the IVS, the dependent source also gets turned off and its effect is not taken into account.

Part V

Computing this directly would be quite involved. However, we have already done all the hard work computing the Thevenin equivalent. Connecting the circuits is the same as



This is a non-inverting op-amp, and therefore

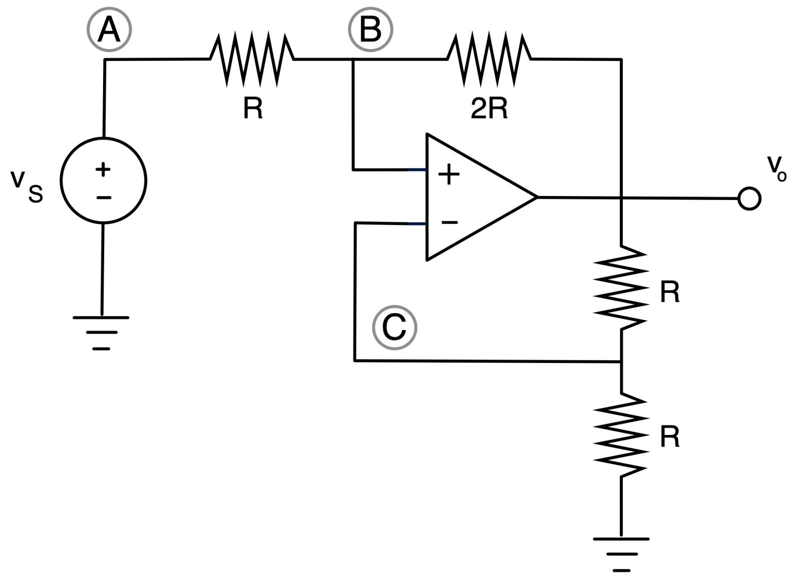
$$V_0 = \frac{R+R}{R} V_T =$$

$$= 2V_T = 2\frac{1+2aR}{2-aR} V_S$$

[+1 extra point]

[+1 extra point for correctly articulating how to use Thevenin equiv. here]

2.-



Part I

As instructed, we use nodal analysis to figure out the output voltage.

We know $V_A = V_S$. [+1 point]

KCL at node B gives us (with $i_p = 0$)

$$\frac{1}{R}(V_B - V_S) + \frac{1}{2R}(V_B - V_o) = 0$$ [+1 point]

KCL at node C (with $i_n = 0$),

$$\frac{1}{R}(V_C - V_o) + \frac{1}{R}(V_C) = 0$$ [+1 point]

From ideal conditions, we have

$$V_B = V_C$$ [+1 point]

We have 3 eqs. in 3 unknowns V_B, V_C, V_O, so we can solve. From the 2nd equation,

$$V_O = 2V_C = 2V_B$$

Substituting into the 1st equation,

$$\frac{1}{R}(V_B - V_S) + \frac{1}{2R}(V_B - 2V_B) = 0 \Rightarrow V_B = 2V_S$$

Therefore

$$V_O = 2V_B = 4V_S$$ [+1 point]

Part II

When the engineer connects the load resistor $R_L = 10\Omega$, the voltage drop this resistor sees is V_O.

If the connected source is $V_S = 3V$, then $4 \cdot V_S = 4 \cdot 3 = 12 > 10V$, so the op-amp gets saturated and $V_O = 10V$. [+1 point]

Therefore, the power delivered to the load is $P_L = V_O^2 \cdot 1/R_L = 100/10 = 10W$ [+1 point]

The range of values for the voltage source so that the op-amp operates linearly is

$$-10V < V_0 = 4V_S < 10V$$

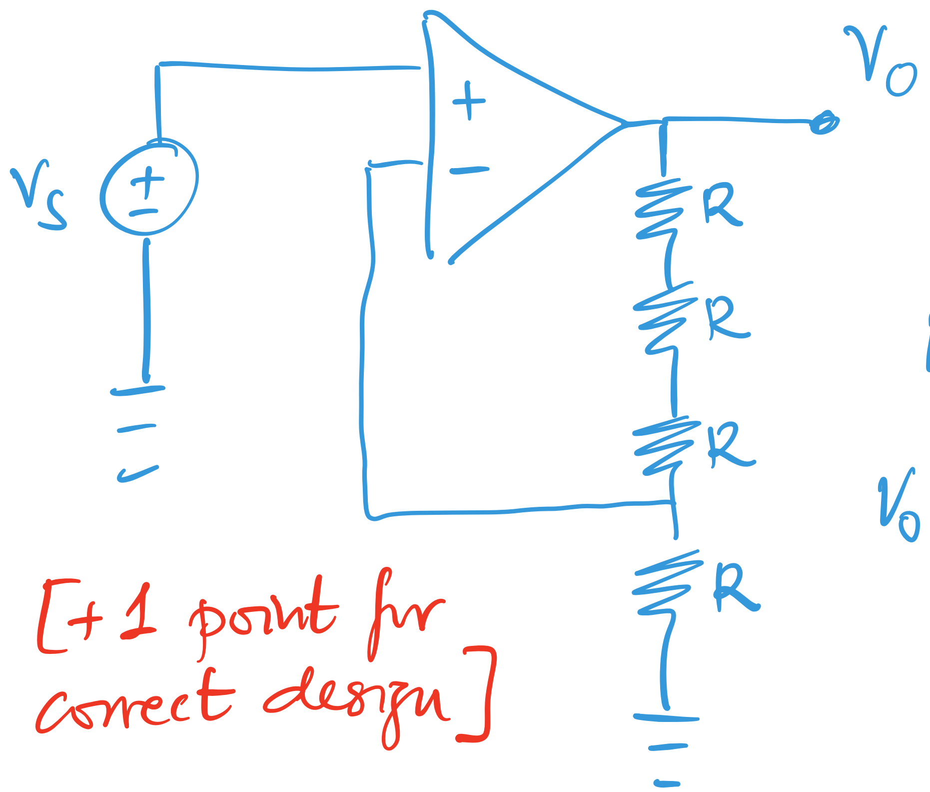
$$-2.5V < V_S < 2.5V$$

[+1 point]

Part III

Since we want to design a circuit with $V_0 = 4V_S$, we use a non-inverting op-amp.

[+1 point]



[+1 point for correct design]

With this, we have

$$V_0 = \frac{3R+R}{R} V_S = $$

$$= 4V_S$$