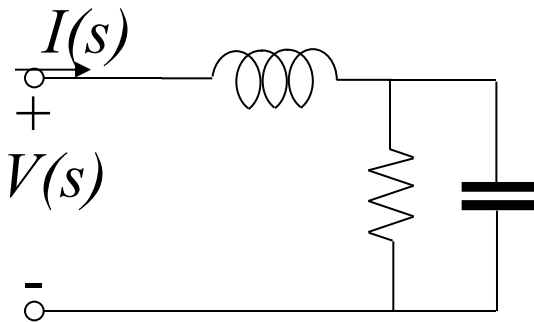


# Designing Circuits – Synthesis - Lego

Port = a pair of terminals to a cct

One-port cct; measure  $I(s)$  and  $V(s)$  at same port

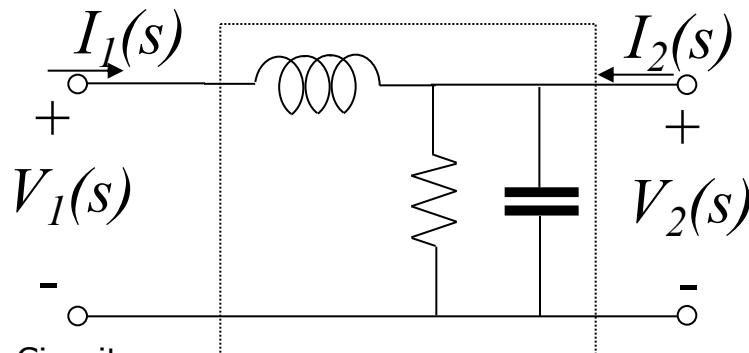


$$Z(s) = \frac{V(s)}{I(s)} = sL + \frac{1}{R^{-1} + sC}$$

Driving-point impedance = input impedance = equiv impedance =  $Z(s)$

Two-ports

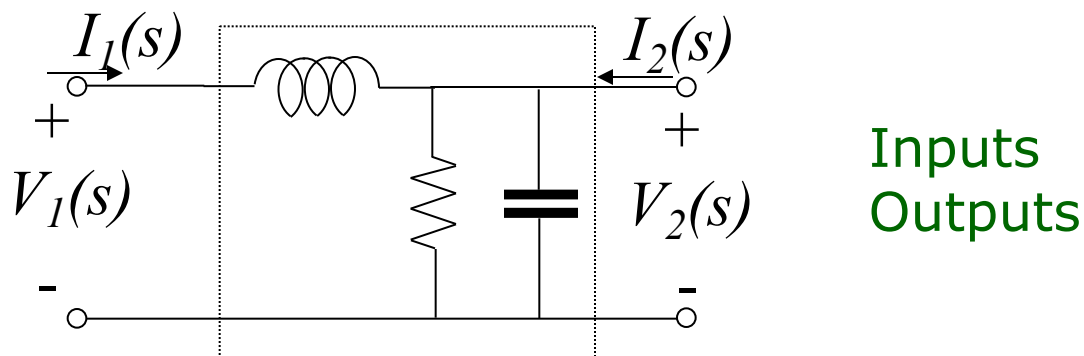
Transfer function; measure input at one port, output at another



Inputs  
Outputs

# Transfer functions

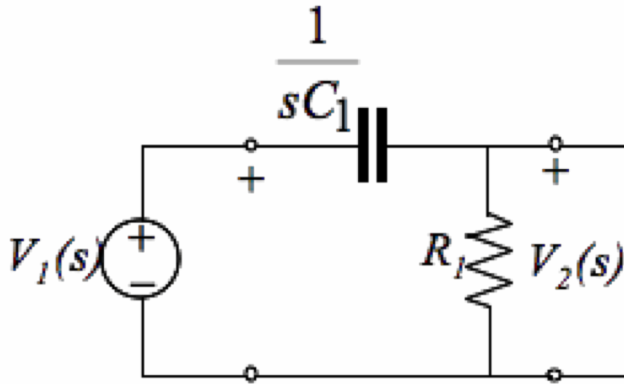
Transfer function; measure input at one port, output at another



$$\text{Transfer function} = \frac{\text{zero - state response transform}}{\text{input signal transform}}$$

*(I.e., what the circuit does to your input)*

## Example 11-2, T&R, 6th ed



Transfer function?

Input impedance?

$$T_v(s) = \frac{V_2(s)}{V_1(s)} = \frac{R_1}{R_1 + \frac{1}{sC_1}} = \frac{s}{s + \frac{1}{R_1C_1}}$$

$$Z(s) = \frac{V_1(s)}{I_1(s)} = R_1 + \frac{1}{sC_1}$$

# Cascade Connections

We want to apply a chain rule of processing

$$T_V(s) = T_{V1}(s) \times T_{V2}(s) \times T_{V3}(s) \times \dots \times T_{Vk}(s)$$

When can we do this by cascade connection of OpAmp ccts?

*Cascade* means output of  $cct_i$  is input of  $cct_{i+1}$

This makes the design and analysis much easier

This rule works if stage  $i+1$  does not load stage  $i$

Voltage is not changed because of next stage

Either

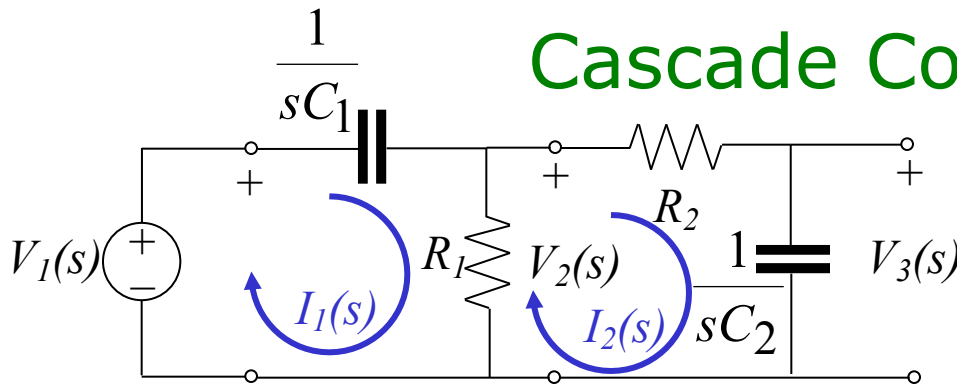
Output impedance of source stage is zero

Or

Input impedance of load stage is infinite

Works well if  $Z_{out,source} \ll Z_{in,load}$

# Cascade Connections



Is chain rule valid?

$$T_{\text{total}}(s) = T_{v_1}(s) \times T_{v_2}(s) = \frac{R_1 C_1 s}{R_1 C_1 s + 1} \times \frac{1}{R_2 C_2 s + 1} = \frac{R_1 C_1 s}{(R_1 R_2 C_1 C_2) s^2 + (R_1 C_1 + R_2 C_2) s + 1}$$

Mesh analysis

$$\begin{aligned} \left( \frac{1}{sC_1} + R_1 \right) I_1(s) - R_1 I_2(s) &= V_1(s) \\ -R_1 I_1(s) + \left( R_1 + R_2 + \frac{1}{sC_2} \right) I_2(s) &= 0 \end{aligned} \quad \begin{pmatrix} I_1(s) \\ I_2(s) \end{pmatrix} = \begin{pmatrix} \frac{1}{sC_1} + R_1 & -R_1 \\ -R_1 & \frac{1}{sC_2} + R_1 + R_2 \end{pmatrix}^{-1} \begin{pmatrix} V_1(s) \\ 0 \end{pmatrix}$$

$$I_2(s) = \frac{s^2 C_1 C_2 R_1}{(R_1 R_2 C_1 C_2) s^2 + (C_1 R_1 + C_2 R_1 + C_2 R_2) s + 1} V_1(s)$$

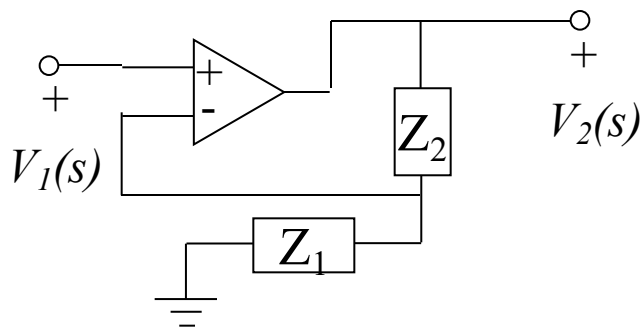
$$V_3(s) = \frac{1}{sC_2} I_2(s) = \frac{R_1 C_1 s}{(R_1 R_2 C_1 C_2) s^2 + (R_1 C_1 + R_1 C_2 + R_2 C_2) s + 1} V_1(s) \quad \text{No! Why?}$$

# Cascade Connections – OpAmp ccts

OpAmps can be used to achieve the chain rule property for cascade connections

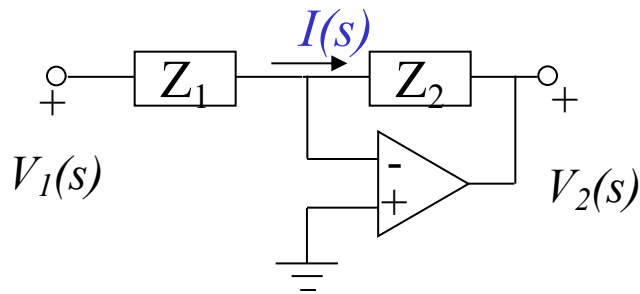
The input to the next stage needs to be driven by the OpAmp output

Consider standard configurations



Noninverting amplifier

No current drawn from  $V_1$  – no load

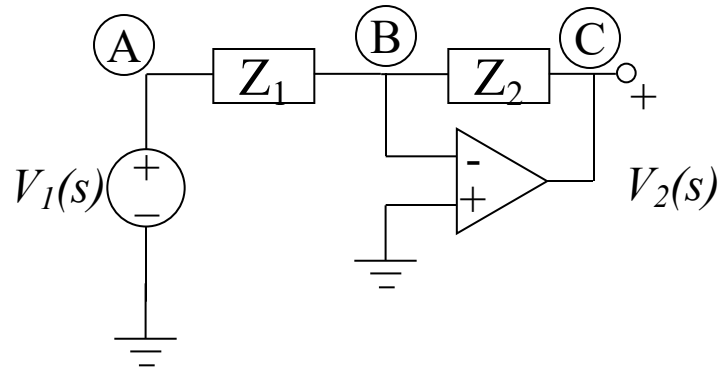


Inverting amplifier

Current provided by  $V_1(s)$  
$$I(s) = \frac{V_1(s)}{Z_1(s)}$$

Need to make sure that stage is driven by OpAmp output to avoid loading  $V_1(s)$

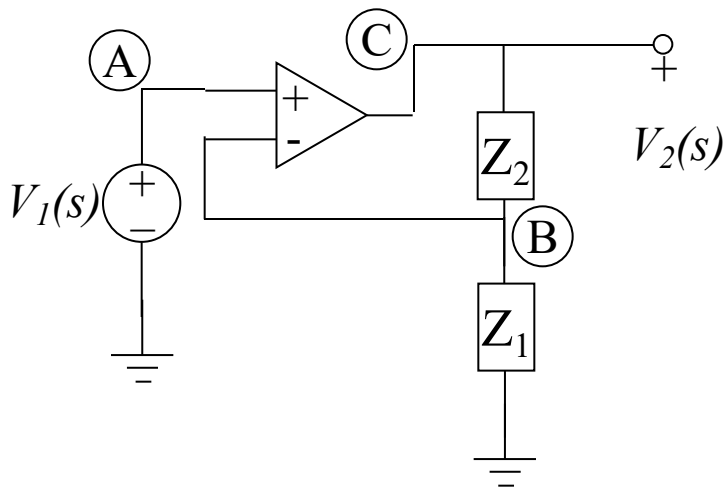
# OpAmp Ccts and transfer functions



Node B:

$$\frac{V_B(s) - V_1(s)}{Z_1(s)} + \frac{V_B(s) - V_2(s)}{Z_2(s)} = 0$$

$$V_B(s) = 0 \Rightarrow T_V(s) = \frac{V_2(s)}{V_1(s)} = -\frac{Z_2(s)}{Z_1(s)}$$



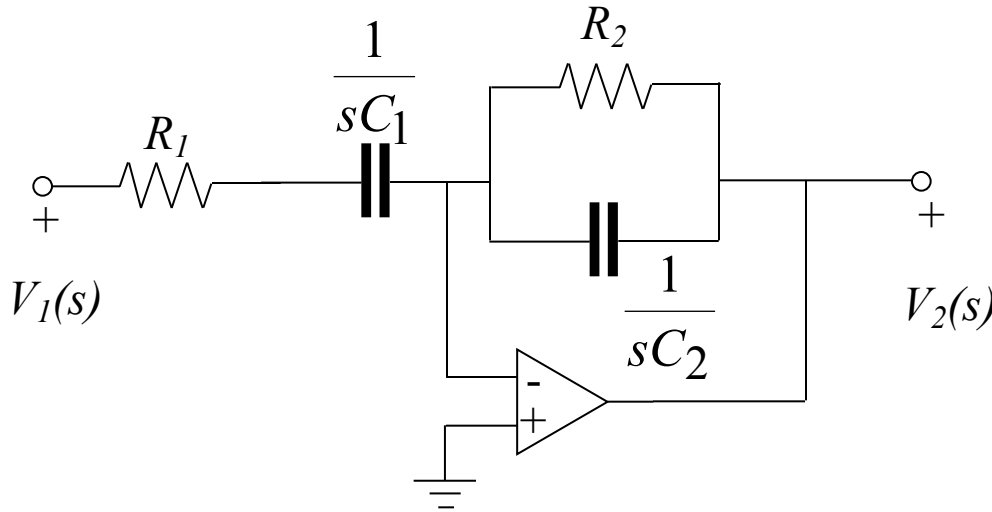
Node B:

$$\frac{V_B(s) - V_2(s)}{Z_2(s)} + \frac{V_B(s)}{Z_1(s)} = 0$$

$$V_B(s) = V_1(s) \Rightarrow T_V(s) = \frac{Z_1(s) + Z_2(s)}{Z_1(s)}$$

# Example 11-4, T&R, 5th ed, p511

Find the transfer function from  $V_1(s)$  to  $V_2(s)$



$$Z_1(s) = R_1 + \frac{1}{sC_1} = \frac{sR_1C_1 + 1}{sC_1}$$

$$Z_2(s) = \frac{R_2 / sC_2}{R_2 + 1/sC_2} = \frac{R_2}{R_2C_2s + 1}$$

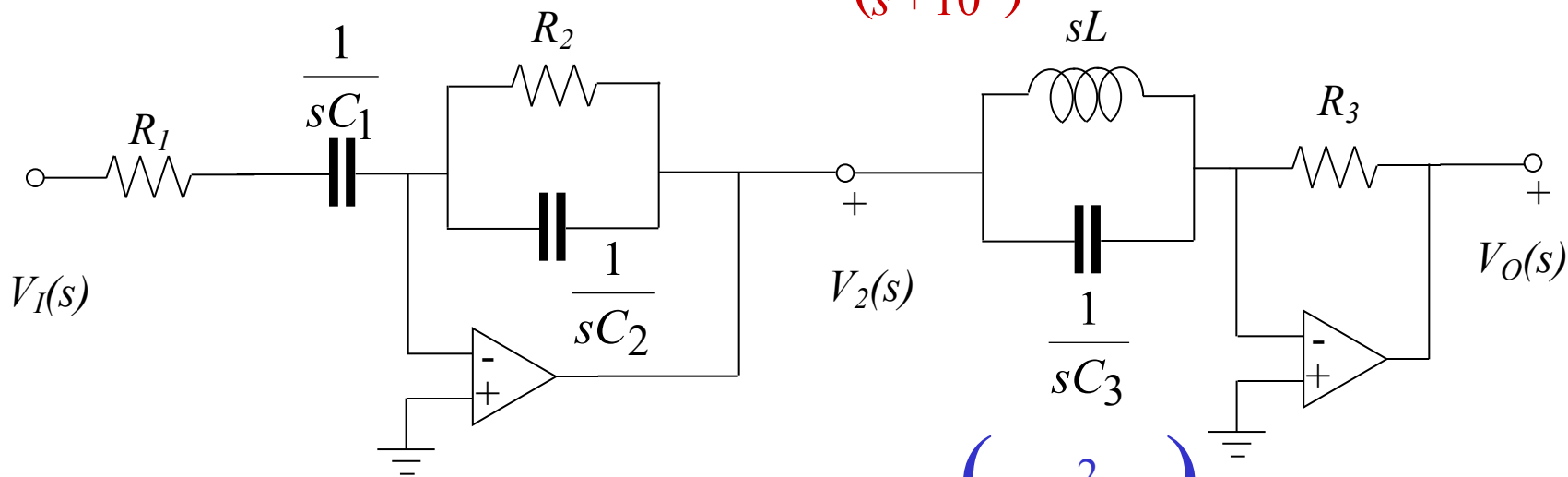
$$T_V(s) = -\frac{sR_2C_1}{(sR_1C_1 + 1)(sR_2C_2 + 1)}$$



# Circuits as Signal Processors

Design a circuit with transfer function

$$\frac{s^2 + 1.42 \cdot 10^5}{s^2 + 2 \cdot 10^4 s + 10^8} = \frac{(s + j 408)(s - j 408)}{(s + 10^4)^2}$$

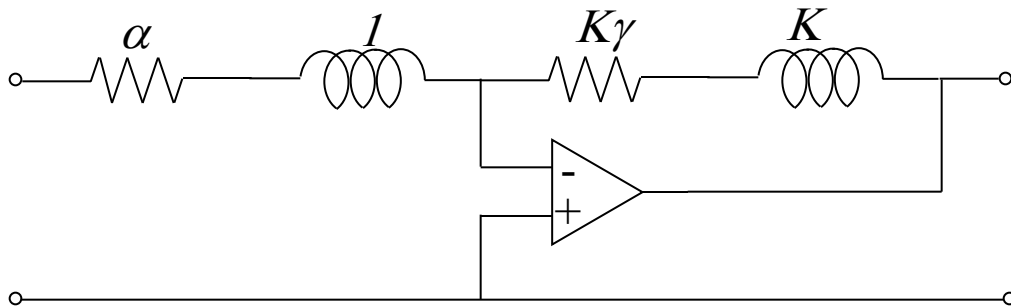


$$T_V(s) = \frac{-sR_2C_1}{(sR_1C_1 + 1)(sR_2C_2 + 1)} \cdot \frac{-R_3(LC_3s^2 + 1)}{Ls}$$

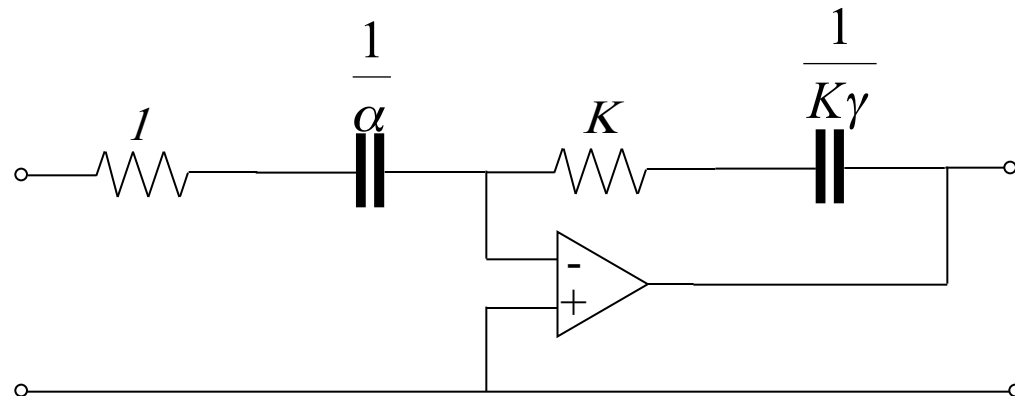
$$R_1 = R_2 = 100\Omega, C_1 = C_2 = 1\mu\text{F}, C_3 = 100\mu\text{F}, L = 70\text{mH}, R_3 = 1\Omega$$

# Transfer Function Design – OpAmp Stages

## First order stages



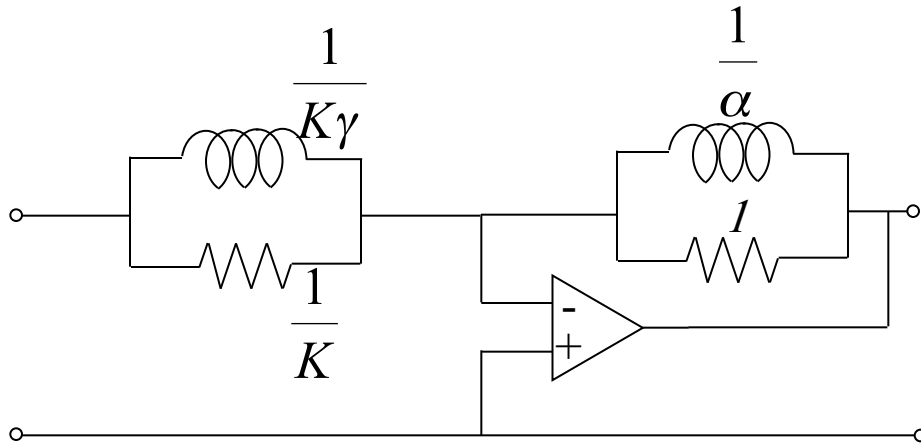
Series RL design



Series RC design

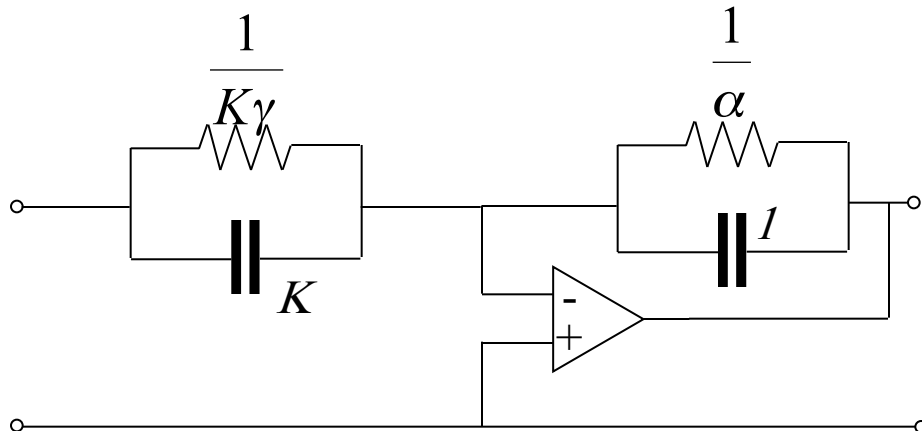
$$T_V(s) = -K \frac{s + \gamma}{s + \alpha}$$

# First-order stages



Parallel RL design

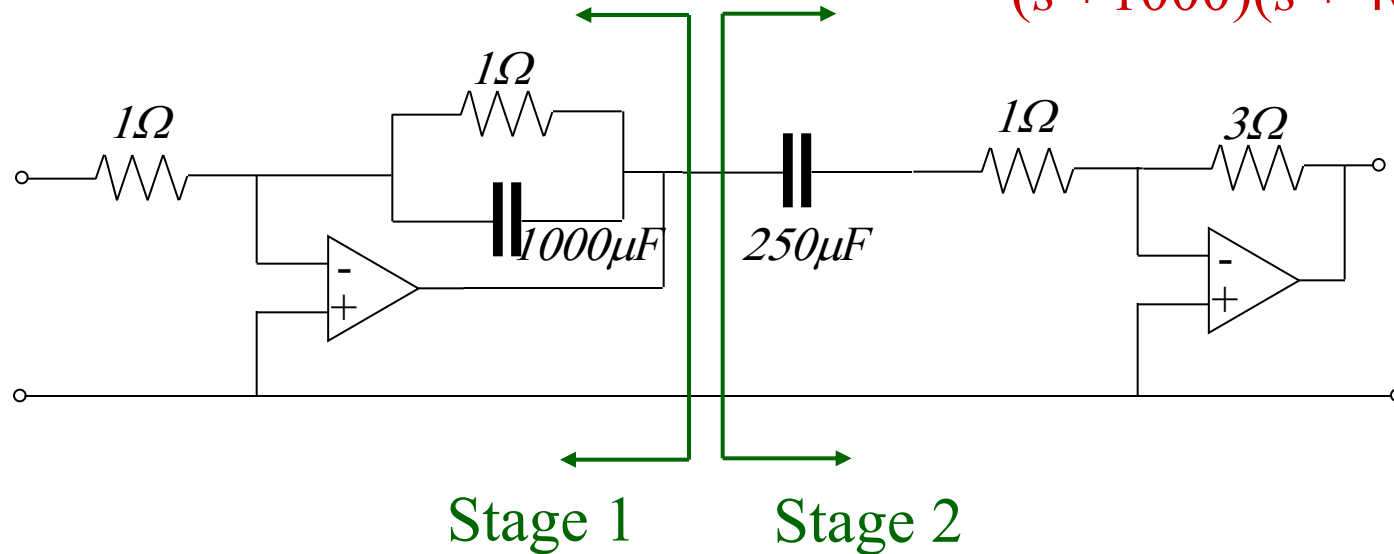
$$T_V(s) = -K \frac{s + \gamma}{s + \alpha}$$



Parallel RC design

# Design Example 11-20, T&R, 5th ed, p 542

Design two ccts to realize  $T_V(s) = \frac{3000s}{(s+1000)(s+4000)}$

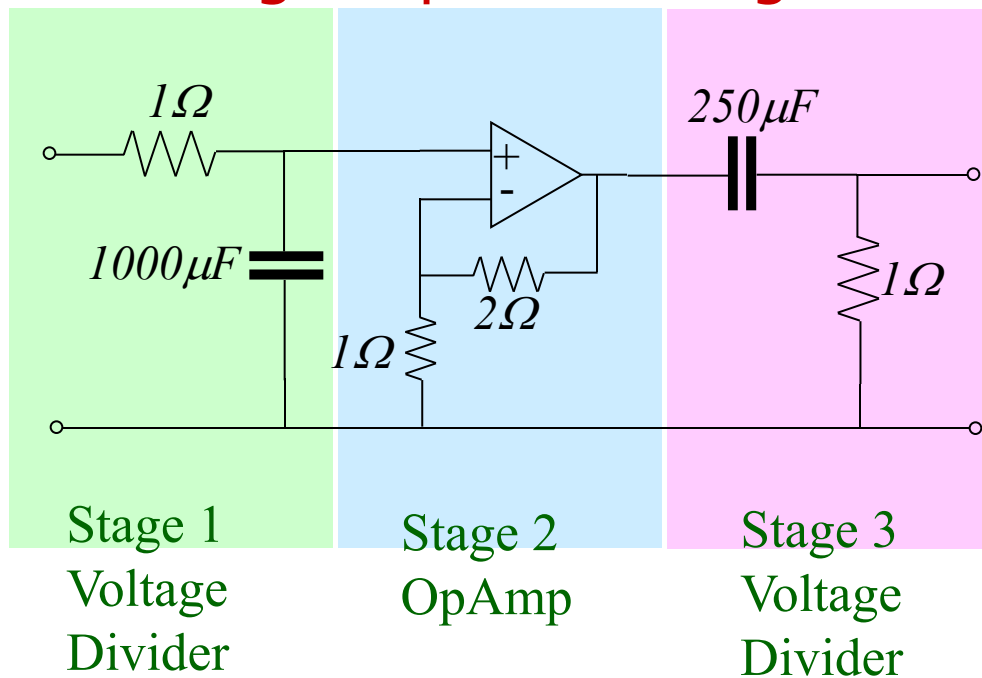


$$T_{V1}(s) = - \left[ \frac{1 / 10^{-3} s}{1 + 1 / 10^{-3} s} \right] [1]^{-1} = \frac{-1000}{s + 1000} \quad T_{V2}(s) = -[3] \left[ 1 + \frac{4000}{s} \right]^{-1} = \frac{-3s}{s + 4000}$$

Unrealistic component values – scaling needed

# Design Example 11-19, T&R, 5th ed, p 539

Non-inverting amplifier design  $T_V(s) = \frac{3000s}{(s+1000)(s+4000)}$



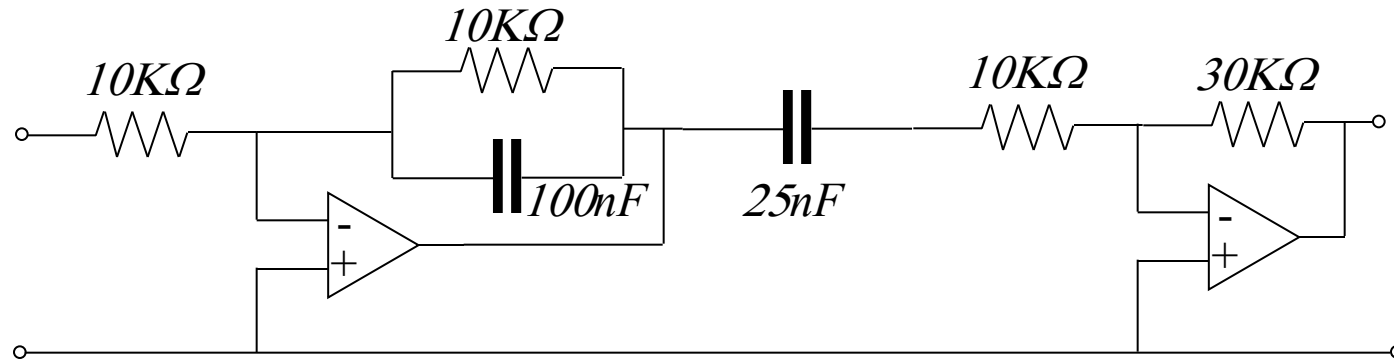
Less OpAmps but more difficult design

Three stage: last stage not driven

Unrealistic component values still – scaling needed

# Scaled Design Example 11-21, T&R, 5th ed, p 544

## More realistic values for components



Need to play games with elements to scale

The ratio formulas for  $T_V$  help permit this scaling

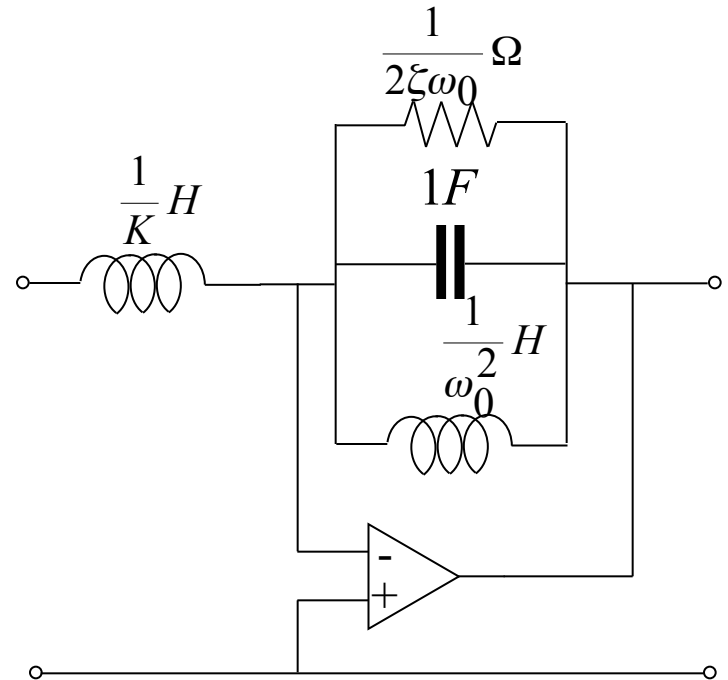
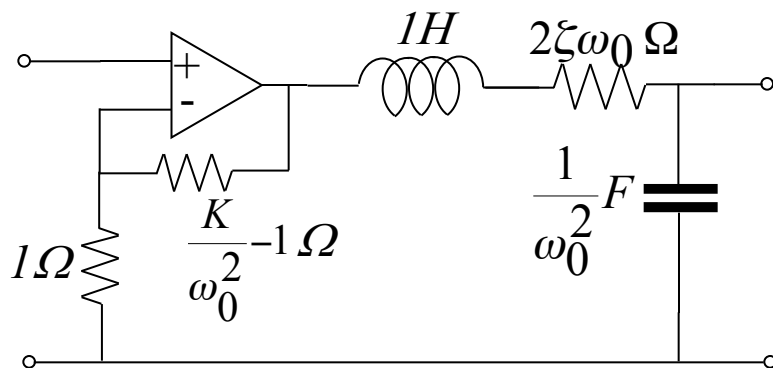
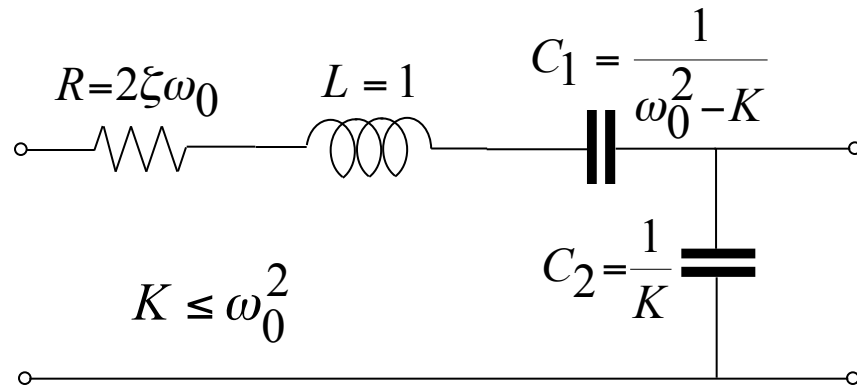
It certainly is possible to demand a design  $T_V$  which is unrealizable with sensible component values

Like a pole at  $10^{-3}$  Hz

# Second-order Stage Design

Circuit stages to yield

$$T_v(s) = \frac{K}{s^2 + 2\zeta\omega_0 s + \omega_0^2}$$



# Circuit Synthesis

Given a stable transfer function  $T_V(s)$ , realize it via a cct using first-order and second-order stages

$$T_V(s) = \frac{\alpha s^2 + \beta s + \gamma}{as^2 + bs + c}$$

$$T_V(s) = \frac{\alpha s + \beta}{as + b}$$

We are limited to stable transfer functions to keep within the linear range of the OpAmps

There is an exception

When the unstable  $T_V(s)$  is part of a stable feedback system

Come to MAE143B to find out

Transistor cct design is conceptually similar