MAE40 - Linear Circuits - Winter 25
Final Exam, March 20, 2025

Instructions

(i) You can use 3 two-sided 1-page handwritten cheatsheets.
(ii) The exam has 4 questions and 1 bonus question, for a total of 40 points and 5 bonus points.
(iii) You have from 3:00pm to 6:00pm to do the exam – but should require less time!
(iv) You can use a calculator with no communication capabilities.
(v) In your responses, clearly *articulate your reasoning* and *properly justify the steps.*
(vi) **Important:** start each part below on a separate page, and write your name & PID at the top of each page.

Good luck!

1. s**-Domain Analysis**

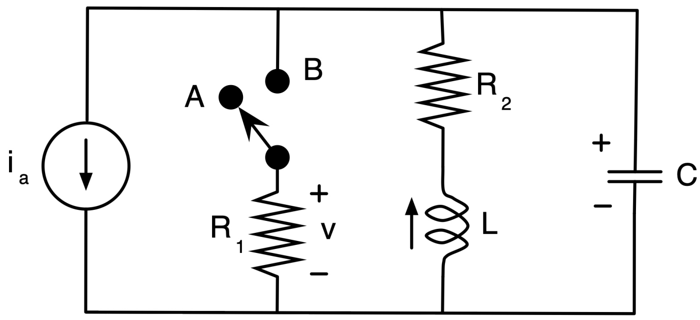


Figure 1: Circuit for Question 1.

Consider the circuit depicted in Figure 1. The value i_a of the current source is constant. The switch is kept in position **A** for a very long time. At time $t = 0$, it is moved to position **B**.

Part I: [3 points] Find the initial condition $v_C(0)$ for the capacitor and $i_L(0)$ for the inductor.

Part II [4 points] Transform the circuit in Figure 1 into the s-domain, using current sources to account for the initial conditions.

Part III [3 points] Set up node-voltage equations for the transformed circuit obtained in Part II (write them in matrix form) and express the transform of the R_1-resistor voltage in terms of the node voltages. No need to solve any equations!

2. **Laplace Domain Circuit Analysis**

Part I: [1 point] Consider the circuit depicted in Figure 2. The value v_A of the voltage source at the top is constant. The switch is kept in position **A** for a very long time. At $t = 0$ it is moved to position **B**. Show that the initial condition for the inductor is given by

$$i_L(0^-) = -\frac{v_A}{R}.$$

[Show your work]

Part II: [4 points] Use this initial condition to transform the circuit into the s-domain for $t \geq 0$. Use an equivalent model for the inductor in which the initial condition appears as a voltage source. Do you recognize the resulting circuit as one of the basic op-amp building blocks? Express the output response transform $V_o(s)$ as a function of $V_i(s)$ and v_A.

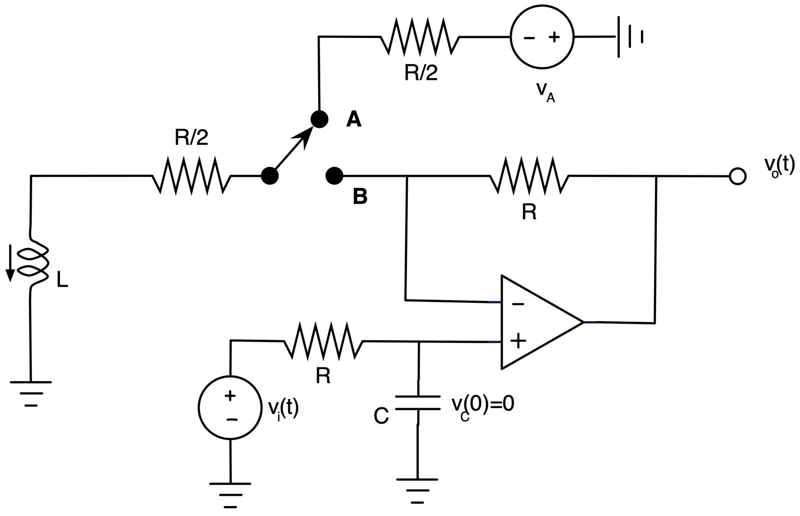


Figure 2: RCL circuit for Laplace Analysis for Question 2.

Part III: [3 points] Use partial fractions and inverse Laplace transforms to show that the output voltage $v_o(t)$ when $v_A = 1\,V$, $v_i(t) = e^{-1500t}u(t)\,V$, $C = 1$ mF, $L = 1$ mH, and $R = 1\,\Omega$ is

$$v_o(t) = (e^{-500t} - 2e^{-1000t})u(t).$$

Part IV: [2 points] Decompose the output voltage of Part III as (i) the sum of the zero-state and zero-input response, and (ii) the sum of the natural and forced response.

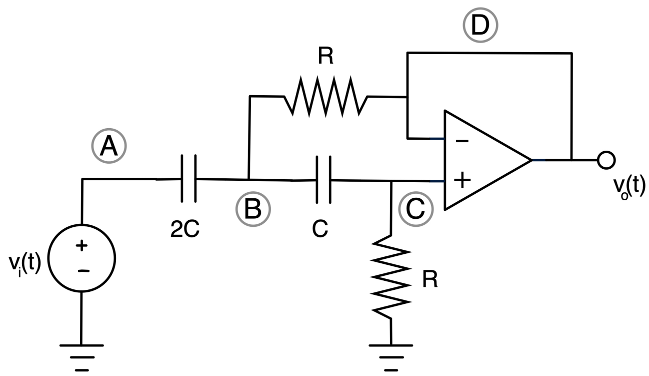


Figure 3: Active Filter Circuit for Question 3.

3. **Active Filter Analysis and Frequency Response**

Part I: [1 point] Assuming zero initial conditions, transform the circuit in Figure 3 into the s-domain.

Part II: [3 points] Use nodal analysis to show that the transfer function from $V_i(s)$ to $V_o(s)$ is

$$T(s) = \frac{V_o(s)}{V_i(s)} = \frac{s^2}{s^2 + (3\lambda/2)s + \lambda^2/2}$$

where $\lambda = 1/(RC)$.

Part III: [2 points] Compute the gain function $|T(j\omega)|$ and the phase function $< T(j\omega)$

Part IV: [2.5 points] What are the DC gain and the ∞-freq gain? What are the corresponding values of the phase function? What is the gain at $\omega = \lambda$?

Part V: [1.5 points] What type of filter is this circuit? Is the cut-off frequency of the circuit equal, less or greater than λ? Sketch the gain versus frequency diagram.

4. **Active Filter Design**

Consider the transfer function

$$T(s) = \frac{V_0(s)}{V_i(s)} = \frac{s^2}{s^2 + (3\lambda/2)s + \lambda^2/2}$$

of the filter studied in the previous question where λ is now to be specified by the user. In this question, always assume zero initial conditions.

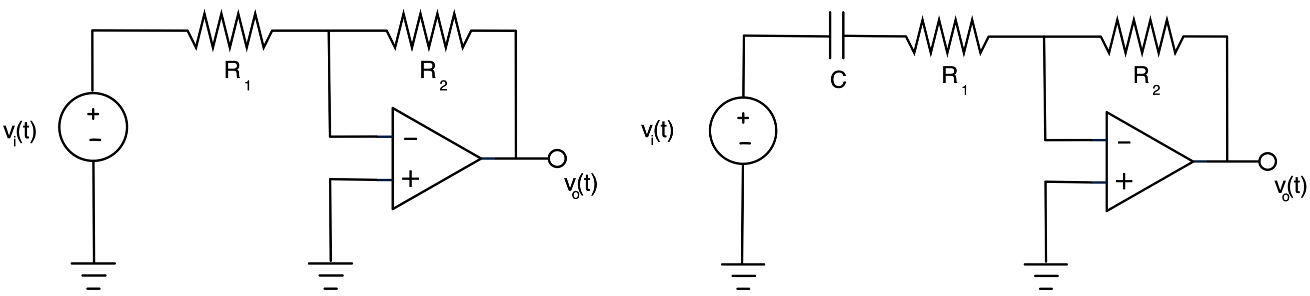
Part I: [4 points] Show that the transfer function $T(s)$ can be realized as a product of two first-order high-pass filters of the form

$$T_1(s) = \frac{\pm s}{s + \omega_1}, \qquad T_2(s) = \frac{\pm s}{s + \omega_2}$$

that is, $T(s) = T_1(s) \times T_2(s)$. What is the cut-off frequency and maximum gain of $T_1(s)$ and $T_2(s)$ in terms of λ?

Part II: [3 points] Design a circuit that implements $T(s)$ as the product of $T_1(s)$ and $T_2(s)$ using 2 OpAmps in total.

Part III: [3 points] Design a circuit that implements $T(s)$ as the product of $T_1(s)$ and $T_2(s)$ using 1 OpAmp in total.



(a) Circuit I (b) Circuit II

Figure 4: OpAmp Circuits for AC Amplifier in Question 5.

5. **Bonus question: AC Amplifier**

A group of former MAE 40 students needs to build an OpAmp circuit to amplify telephone signals. Because of reasons that were relevant some hundred years ago, your telephone jack has a DC 48 V voltage superimposed to the voice signal. To make things simple, think now of a "test" signal coming through your telephone line with a fixed frequency of $\omega = 1500$ rad/s. On your telephone jack you would measure a voltage

$$v_i(t) = 48 + 12\cos(\omega\, t) \text{ V}.$$

In order to solve this problem the students have selected two circuits, shown in Figure 4. The power supplies (not shown in the figures) to the OpAmps are $V_{CC} = +12$ V and $-V_{CC} = -12$ V. For impedance matching set $R_1 = 600\ \Omega$.

Part I: [Extra 1 point] The students built Circuit I with $R_1 = R_2 = 600\ \Omega$ but found out to their surprise that they could not hear the $\omega = 1500$ rad/s tone after they plugged $v_o^{SS}(t)$ into a speaker. Why did that happen?

(**Hint:** Recall the role played by V_{CC} and $-V_{CC}$ in an OpAmp!)

Part II: [Extra 2 points] By replacing R_2 with a potentiometer, they eventually found a value of resistance R_2 below which they could hear the tone *without any distortion*. What is this critical value of R_2 and what is the corresponding amplitude of the tone fed to the speaker?

Part III: [Extra 2 points] One student suggested that all they needed to do to fix the problem was to add a capacitor to Circuit I, obtaining Circuit II. In order to explain why this circuit also solves the problem, compute the steady-state response $v_o^{SS}(t)$ as a function of C (with $R_1 = R_2 = 600\ \Omega$). Why does this circuit fix the problem?