Linear Functional Observers for Unforced Multi-Output Nonlinear Systems

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Abstract: This paper focuses on the problem of designing linear functional observers for unforced multi-output nonlinear systems. Existence conditions of linear functional observers for a general nonlinear system are provided. The given conditions are necessary and sufficient for the estimation of a single function of states for multi-output systems using a functional observer with linear dynamics and a linear output map. Observer structure for a system satisfying the conditions mentioned is also given. The theoretical results are confirmed by computer simulation.

Keywords: Linear functional observer, estimation, nonlinear systems, unforced multi-output systems

1. INTRODUCTION

In many practical applications, it is not possible to measure the states of the system. In those cases, we need an observer to estimate the system states. Most of the times, rather than the whole state vector, only a function of the states is required to be observed or estimated. So, instead of going for a full order or reduced order observer, we can go for the design of a functional observer which will estimate only the required function of states.

For linear systems, functional observers have been designed in the past with the first major result given by Luenberger (1971). Since then, considerable work has been done (Aldeen and Trinh, 1999; Trinh and Fernando, 2012). Mostly, the emphasis has been on designing the observer with the minimum possible order (Fernando et al., 2010). But for nonlinear systems, especially for the cases when the function to be estimated is also a nonlinear function of the states, not much work has been done. In Tami et al. (2013), there has been efforts to do this using the concepts of Z-observability and Partial Observability Normal Form (PONF). There has been an effort to generalize Luenberger's result for linear systems to nonlinear systems in Kravaris (2011). But none of the works done so far considers the design of functional observers for multi-output nonlinear systems with the required function to be estimated also being a nonlinear function of the states. For nonlinear systems variety of methos and approaches of exact linearization is available in literature (Kazantzis and Kravaris, 1998; Kazantzis et al., 2000; Krener and Xiao, 2002, 2001) .

The existing result provides a direct generalization of Luenberger's linear theory of the functional observer for nonlinear systems in Kravaris (2016). Inspired by the above discussion, this paper tried to design functional observers for multi-output unforced nonlinear systems. We have specifically focused on designing linear functional observers for the estimation of a non-linear function of states in nonlinear systems. We made use of the Lyapunov's auxiliary theorem mentioned in Lyapunov (1992) to obtain the conditions that guarantee the existence of solutions of a system of partial differential equations.

1.1 Main Contributions

The main contributions of this paper are as follows:

(1) The problem of designing linear functional observers for multi-output nonlinear systems has been considered with . Existence conditions of linear functional observers for a general nonlinear system are provided.
(2) The necessary and sufficient conditions are given for the estimation of a single function of states for multi-output systems using a functional observer with the linear dynamics and a linear output map.
(3) The theoretical results are confirmed by computer simulation result.

1.2 Organization of paper

This paper is organized as follows. Section 2 mentions the problem statement. Section 3 provides the existence conditions of a functional observer for a general nonlinear system obtained from the direct generalization of Luenberger's condition for linear systems and also gives the structure of the observer if the dynamics and the output map are constrained to be linear. In section 4, conditions for the existence of a linear functional observer are given. Section 5 gives the structure of the observer if the conditions mentioned in section 4 are satisfied. Section 6 contains an example to show the effectiveness of the above algorithm. Finally, section 7 concludes the paper.

2. PROBLEM STATEMENT

Consider an unforced multi-output nonlinear system described by

$$\begin{aligned} \frac{dx}{dt} &= f(x) \\ y &= h(x) \\ z &= q(x) \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state vector, $y \in \mathbb{R}^p$ is the output vector and $z \in \mathbb{R}$ is the desired nonlinear function to be estimated and $f : \mathbb{R}^n \to \mathbb{R}^n$, $h : \mathbb{R}^n \to \mathbb{R}^p$ and $q : \mathbb{R}^n \to \mathbb{R}$ are analytic functions. We are required to estimate the function z with the measurement of the outputs y without estimating the state vector x. Instead of assuming that the system is observable like the normal observer design, in case of functional observers, we need not have that condition fulfilled in every case. The only assumption, which we need to make is that the function which we are required to estimate should lie in the span of the observability matrix.

The problem that we have considered in this paper is to design a linear functional observer of order $v(\leq n-p)$ to estimate z.

3. SOLUTION APPROACH

3.1 Generalization of Luenberger's definition to nonlinear systems

Generalizing Luenberger's functional observer to nonlinear system, the structure of the observer comes out to be

$$\frac{d\hat{\xi}}{dt} = \varphi(\hat{\xi}, y) \tag{2}$$

$$\hat{z} = \omega(\hat{\xi}, y) \tag{3}$$

where $\hat{\xi} \in \mathbb{R}^v$ is the observer state vector, $\hat{z} \in \mathbb{R}$ is the observer output, $\varphi : \mathbb{R}^v \times \mathbb{R}^p \to \mathbb{R}^v$ and $\omega : \mathbb{R}^v \times \mathbb{R}^p \to \mathbb{R}$. Existence of a mapping

$$\xi = \theta(x) = \begin{bmatrix} \theta_1(x) \\ \vdots \\ \theta_v(x) \end{bmatrix} \tag{4}$$

from $\mathbb{R}^n$ to $\mathbb{R}^v$ under the following conditions guarantee the existence of a functional observer of v^{th} order

$$\frac{\partial\theta(x)}{\partial x} f(x) = \varphi(\theta(x), h(x)) \tag{5}$$

$$q(x) = \omega(\theta(x), h(x)) \tag{6}$$

3.2 Linearization of the functional observer

Constraining the dynamics and the output equation of the above mentioned generalized functional observer to be linear, (2) and (3) would be given by

$$\frac{d\hat{\xi}}{dt} = A\hat{\xi} + By \tag{7}$$

$$\hat{z} = C\hat{\xi} + Dy \tag{8}$$

where A, B, C, D are $v \times v, v \times p, 1 \times v, 1 \times p$ matrices respectively. Then, the error dynamics of the observer comes out to be

$$\frac{d}{dt}(\hat{\xi} - \theta(x)) = A(\hat{\xi} - \theta(x)) \tag{9}$$

which are stable if matrix A is chosen to be stable. But we have to make sure that conditions (5) and (6) are satisfied, i.e. we should be able to solve the system of partial differential equations

$$\frac{\partial\theta(x)}{\partial x} f(x) = A\theta(x) + Bh(x) \tag{10}$$

and then are able to express $q(x)$ as

$$q(x) = C\theta(x) + Dh(x) \tag{11}$$

Considering equation (10), the left hand side has matrix $\dfrac{\partial\theta(x)}{\partial x}$ with dimension $v \times n$ and $f(x)$ with dimension $n \times 1$. If we take the n elements of $f(x)$ as $f_i(x)$ for $(i = 1, \ldots, n)$, then equation (10) would be given by

$$\begin{bmatrix} \sum_{i=1}^{n} \frac{\partial\theta_1(x)}{\partial x_i} f_i(x) \\ \vdots \\ \sum_{i=1}^{n} \frac{\partial\theta_v(x)}{\partial x_i} f_i(x) \end{bmatrix} = \tag{12}$$

$$\begin{bmatrix} A_{11}\theta_1(x) \ldots + A_{1v}\theta_v(x) + B_{11}h_1(x) + \ldots + B_{1p}h_p(x) \\ \vdots \\ A_{v1}\theta_1(x) \ldots + A_{vv}\theta_v(x) + B_{v1}h_1(x) + \ldots + B_{vp}h_p(x) \end{bmatrix} \tag{13}$$

where A_{ij}s $(i, j = 1, ..., v)$ and B_{ij}s $(i = 1, \ldots, v, \quad j = 1, \ldots, p)$ are the elements of the A and B matrix, respectively.

4. EXISTENCE CONDITIONS OF THE LINEAR FUNCTIONAL OBSERVER

The conditions for the solvability of the system of partial differential equations (12) could be obtained from Lyapunov's auxiliary theorem (Lyapunov, 1992; Kazantzis and Kravaris, 1997).

4.1 Lyapunov's Auxiliary Theorem

Let there be given a system of partial differential equations

$$\sum_{s=1}^{n} (p_{s1}x_1 + p_{s2}x_2 + \ldots + p_{sn}x_n + X_s)\frac{\partial z_j}{\partial x_s} = q_{j1}z_1 + q_{j2}z_2 + \ldots + q_{jk}z_k + Z_j \quad (j = 1, 2, \ldots, k) \tag{14}$$

where $X_1, X_2, \ldots, X_n, Z_1, Z_2, \ldots, Z_k$ are holomorphic functions of the variables $x_1, x_2, \ldots, x_n, z_1, z_2, \ldots, z_k$ becoming zero when all these variables become zero. We assume: that the functions X_s do not contain in their expansions terms of the first degree; that the terms of first degree appearing in the functions Z_j do not depend on the quantities $z_1, z_2, \ldots, z_k$; that the $p_{s\sigma}, q_{jl}$ are constants, such that $\chi_1, \chi_2, \ldots, \chi_n$ being the roots of the equation

$$\begin{vmatrix} p_{11}-\chi & p_{12} & \cdots & p_{1n} \\ p_{21} & p_{22}-\chi & \cdots & p_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ p_{n1} & p_{n2} & \cdots & p_{nn}-\chi \end{vmatrix} = 0 \quad (15)$$

and $\lambda_1, \lambda_2, \dots, \lambda_k$ those of the equation

$$\begin{vmatrix} q_{11}-\lambda & q_{12} & \cdots & q_{1k} \\ q_{21} & q_{22}-\lambda & \cdots & q_{2k} \\ \cdots & \cdots & \cdots & \cdots \\ q_{k1} & q_{k2} & \cdots & q_{kk}-\lambda \end{vmatrix} = 0 \quad (16)$$

the real part of all the χ_s are different from zero and have the same sign, and that, moreover, the numbers χ_s and λ_j are not related by any equation of the form

$$m_1\chi_1 + m_2\chi_2 + \ldots + m_n\chi_n = \lambda_j \quad (j = 1, 2, \dots, k) \quad (17)$$

where all the m_s are non-negative integers satisfying the condition

$$\sum m_s > 0$$

This agreed, we shall always be able to find a system of holomorphic functions $z_1, z_2, \dots, z_k$ of the variables $x_1, x_2, \dots, x_n$, satisfying equations (14) and becoming zero for

$$x_1 = x_2 = \ldots = x_n = 0.$$

Moreover there will be only one such system of functions.

If we compare (14) with (12), then the solvability conditions for (12) are given by:

Condition A

(1) All the eigenvalues of $\frac{\partial f}{\partial x}(0)$ should be nonzero and of same sign.
(2) No eigenvalue λ_j of A should be of the form $\lambda_j = \sum_{i=1}^{n} m_i\chi_i$, where $\chi_i (i = 1, \dots, n)$ are the eigenvalues of $\frac{\partial f}{\partial x}(0)$.

4.2 Necessary and sufficient conditions for the functional observer linearization

Without loss of generality, consider the dynamics of the observer in the following controllable canonical form

$$\frac{d\hat{\xi}}{dt} = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & 1 \\ -\alpha_v & -\alpha_{v-1} & \cdots & -\alpha_1 \end{bmatrix} \hat{\xi} + \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & & & \\ 0 & 0 & \cdots & 0 \\ 1 & k_1 & \cdots & k_{p-1} \end{bmatrix} y$$

This would imply that

$$\frac{\partial \theta(x)}{\partial x} f(x) = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & 1 \\ -\alpha_v & -\alpha_{v-1} & \cdots & -\alpha_1 \end{bmatrix} \theta(x) + \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & & & \\ 0 & 0 & \cdots & 0 \\ 1 & k_1 & \cdots & k_{p-1} \end{bmatrix} \begin{bmatrix} h_1(x) \\ h_2(x) \\ \vdots \\ h_{p-1}(x) \\ h_p(x) \end{bmatrix}$$

which gives the following set of equations

$$\begin{aligned} L_f\theta_1(x) =& \theta_2(x) \\ L_f\theta_2(x) =& \theta_3(x) \\ &\vdots \\ L_f\theta_v(x) =& -\alpha_v\theta_1(x) - \alpha_{v-1}\theta_2(x) - \cdots - \alpha_1\theta_v(x) + \\ & h_1(x) + k_1h_2(x) + \cdots + k_{p-1}h_p(x) \end{aligned} \quad (18)$$

where L_f denotes the Lie derivative along f.

Equation (18) when rewritten and upon repeated differentiation gives the following equations

$$\begin{aligned} L_f^v\theta_1(x) + \alpha_1 L_f^{v-1}\theta_1(x) + \cdots + \alpha_v\theta_1(x) &= h_1(x) + \\ & k_1h_2(x) + \cdots + k_{p-1}h_p(x) \\ L_f^v\theta_2(x) + \alpha_1 L_f^{v-1}\theta_2(x) + \cdots + \alpha_v\theta_2(x) &= L_fh_1(x) + \\ & L_fk_1h_2(x) + \cdots + L_fk_{p-1}h_p(x) \\ & \vdots \\ L_f^v\theta_v(x) + \alpha_1 L_f^{v-1}\theta_v(x) + \cdots + \alpha_v\theta_v(x) &= L_f^{v-1}h_1(x) + \\ & L_f^{v-1}k_1h_2(x) + \cdots + L_f^{v-1}k_{p-1}h_p(x) \end{aligned}$$

If we are able to express $q(x)$ as linear output of the observer, then (11) must be satisfied which gives us

$$\begin{aligned} q(x) =& C_1\theta_1(x) + C_2\theta_2(x) + \cdots + C_v\theta_v(x) + \\ & D_1h_1(x) + D_2h_2(x) + \cdots + D_v\theta_v(x) \end{aligned} \quad (19)$$

Applying the operator $L_f^v + \alpha_1 L_f^{v-1} + \cdots + \alpha_{v-1}L_f + \alpha_v I$ to both sides of the above equation and after simple manipulations, we get

$$\begin{aligned} & L_f^v q(x) + \alpha_1 L_f^{v-1}q(x) + \cdots + \alpha_{v-1}L_fq(x) + \alpha_v q(x) = \\ & D_1L_f^vh_1(x) + (D_1\alpha_1 + C_v)L_f^{v-1}h_1(x) + \cdots + (D_1\alpha_{v-1} \\ & + C_2)L_fh_1(x) + (D_1\alpha_v + C_1)h_1(x) + D_2L_f^vh_2(x) + \\ & (D_2\alpha_1 + C_vk_1)L_f^{v-1}h_2(x) + \cdots + (D_2\alpha_{v-1} + C_2k_1) \\ & L_fh_2(x) + (D_2\alpha_v + C_1k_1)h_2(x) + \cdots\cdots\cdots + \\ & D_pL_f^vh_p(x) + (D_p\alpha_1 + C_vk_{p-1})L_f^{v-1}h_p(x) + \cdots + \\ & (D_p\alpha_{v-1} + C_2k_{p-1})L_fh_p(x) + (D_p\alpha_v + C_1k_{p-1})h_p(x) \end{aligned}$$

which could be written in more compact form as

$$\begin{aligned} \sum_{i=0}^{v} \alpha_i L_f^{v-i} q(x) = \sum_{j=1}^{p} \Big(& D_jL_f^vh_j(x) + \\ & \sum_{i=0}^{v-1} (D_j\alpha_{v-i} + C_{i+1}k_{j-1})L_f^ih_j(x) \Big) \end{aligned} \quad (20)$$

with $\alpha_0 = 1$ and $k_0 = 1$.
This means that for condition (11) to be satisfied for the design of linear functional observer, (20) must be satisfied in addition to the conditions obtained from Lyapunov's Auxiliary theorem for the solvability of the system of partial differential equations (10). This condition could be written as

Condition B We must be able to express $L_f^vq(x) + \alpha_1 L_f^{v-1}q(x) + \cdots + \alpha_{v-1}L_fq(x) + \alpha_vq(x)$ as linear combination of all the outputs and their Lie derivatives upto v^{th} order.

If we examine equation (20) carefully, then it becomes clear that all the coefficients in the linear combination condition are not independent. We have p outputs each with $v+1$ constants. Out of these, only $v+2p-1$ are independent. But, these could also be useful for the observer order reduction than the single output case and even for the existence of the linear functional observer in many cases as would be clear from the example.

5. STRUCTURE OF FUNCTIONAL OBSERVER

Without loss of generality, consider the case when all the coefficients related to $h_1(x)$ and its derivatives are independent and for the rest of the outputs, first two, i.e. those related to $L_f^v h_i(x)$ and $L_f^{v-1} h_i(x)$ are independent. Let the coefficients be β_{ij}s where $i = 1, 2, \cdots, p$ and $j = 0, 1, \cdots, v$ and as per assumption, all β_{1j}s and β_{i0}, β_{i1} are independent. Then the structure of observer with the desired eigenvalues of A matrix comes out to be

$$\dot{\hat{\xi}} = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & 1 \\ -\alpha_v & -\alpha_{v-1} & \cdots & -\alpha_1 \end{bmatrix} \hat{\xi} + \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & & & \\ 0 & 0 & \cdots & 0 \\ 1 & \dfrac{\beta_{21} - \beta_{20}\alpha_1}{\beta_{11} - \beta_{10}\alpha_1} & \cdots & \dfrac{\beta_{p1} - \beta_{p0}\alpha_1}{\beta_{11} - \beta_{10}\alpha_1} \end{bmatrix} y \tag{21}$$

$$\hat{z} = [\beta_{1v} - \alpha_v\beta_{10} \;\; \beta_{1v-1} - \alpha_{v-1}\beta_{10} \;\; \cdots \;\; \beta_{11} - \alpha_1\beta_{10}]\,\hat{\xi} + [\beta_{10} \;\; \beta_{20} \;\; \cdots \;\; \beta_{p0}]\, y \tag{22}$$

The error dynamics of the observer would be given by

$$\dot{e} = \dot{\hat{\xi}} - \dot{\theta}(x) = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & 1 \\ -\alpha_v & -\alpha_{v-1} & \cdots & -\alpha_1 \end{bmatrix} e \tag{23}$$

Based on the above development, the design procedure is now summarized by the Algorithm 1 as follows:

Algorithm 1

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(1) Calculate the eigenvalues of $\dfrac{\partial f}{\partial x}(0)$. All of them should be non-zero and of same sign.
(2) Taking v as 1, check the validity of (20). If it is not satisfied, increase the value of v by 1 and go like this till $(n-p)$. The value of v at which (20) gets satisfied, becomes the order of the functional observer.
(3) Select the v eigenvalues of the A matrix such that condition A(2) is satisfied and also we have the desired degree of stability.
(4) The observer can then be designed using the structure given in Section 5.

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6. NUMERICAL EXAMPLE

Consider the following nonlinear system

$$\begin{aligned} \dot{x}_1 &= -x_1 - x_3^2 + x_1x_2x_3 - \frac{1}{2}x_3^3 \\ \dot{x}_2 &= x_1 - \frac{1}{2}x_3^2 - x_2 \\ \dot{x}_3 &= -x_3 + x_1x_2 - \frac{1}{2}x_3^2 \\ y &= \begin{bmatrix} x_2 \\ x_3^2 \end{bmatrix} \end{aligned}$$

The function which we are required to estimate is given by

$$z = q(x) = x_2 + 2x_1 - x_3^2$$

First of all, checking the eigenvalues of matrix $\dfrac{\partial f}{\partial x}(0)$, i.e. eigenvalues of $\begin{bmatrix} -1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$.

Eigenvalues of this matrix are all three -1, i.e. all are non-zero and of same sign implying that condition $A(1)$ is satisfied. Satisfaction of (20) for $v = 1$ means that a linear functional observer of order one exists and therefore, observer matrix A would be having a single eigenvalue and it should be selected in accordance with condition $A(2)$. Lets take A and hence the eigenvalue of observer matrix as -5.5. Now, we have α_1 as 5.5. According to (21) and (22), the structure of functional observer is given by:

$$\begin{aligned} \dot{\hat{\xi}} &= -5.5\hat{\xi} + [1 \;\; 0.0247]\, y \\ \hat{z} &= -40.5\hat{\xi} + [10 \;\; 0]\, y \end{aligned}$$

The simulation result for the actual and the estimated function value are shown in Fig. 1. The error between the actual and the estimated function is shown in Fig. 2. From these results, it is clear that the observer output follows the required function even in the presence of large initial error.

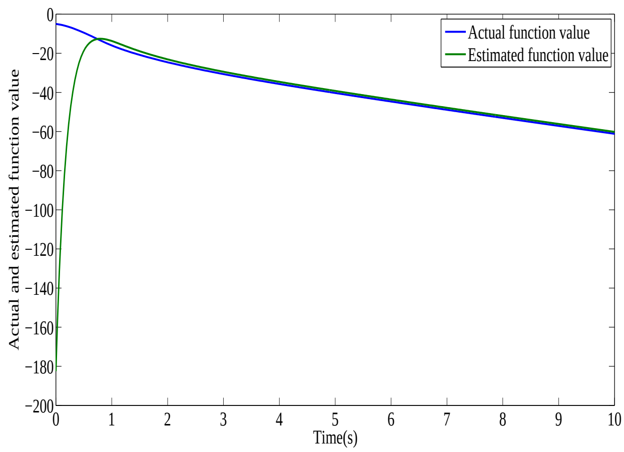

Fig. 1. Actual and the estimated value of the function

7. CONCLUSIONS

In this paper, we have considered the problem of designing a linear functional observer for an unforced multi-output

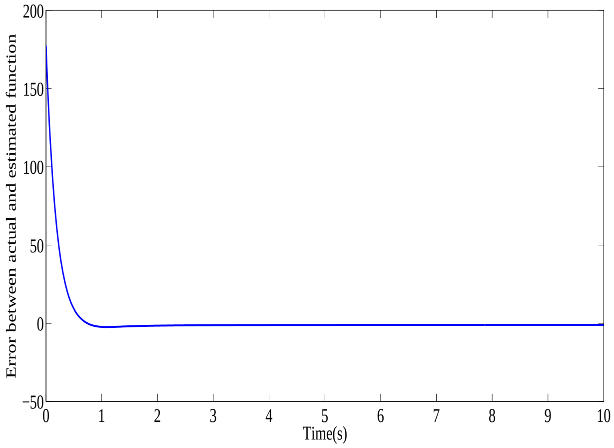

Fig. 2. Error between actual and the estimated function

nonlinear system. The conditions for the existence of the above observer are derived and the structure for the resulting functional observer satisfying those conditions is also proposed. The design procedure and performance of the proposed method are illustrated through a numerical example.

The application of the proposed concept in the various fields, such as sliding mode control (Singh and Sharma, 2012; Singh, 2016; Singh and Janardhanan, 2015), Stochastic systems (Singh and Janardhanan, 2017a; Sharma et al., 2017; Singh and Janardhanan, 2017b) are left for the future.

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