Adaptive Higher Order Sliding Mode Control for Nonlinear Uncertain Systems

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Abstract:
Sliding mode control (SMC) is used for the control of uncertain systems with known uncertainty bounds. But most of the times, the uncertainty bounds are not known. Adaptive SMC (ASMC) is used in these cases for the control of unknown but bounded systems. But ASMC, like the conventional SMC, suffers from chattering. To overcome this drawback, we have proposed an improved control algorithm which uses the concept of adaptive higher-order SMC (AHOSMC). In the proposed algorithm, we have designed the first derivative of the control taking it as the virtual control rather than the actual control itself. The gain-adaptation law does not overestimate uncertainties. The proposed technique has been validated using simulation results.

Keywords: Adaptive higher-order sliding mode control, Nonlinear systems, Tracking, Chattering elimination, Adaptive switching gain law

1. INTRODUCTION

Control under uncertainty conditions is one of the main topics in control theory as in most of the practical cases, system parameters are uncertain. This has lead to intense interest in the development of the so called robust control methods which are supposed to this problem. One particular approach to robust controller design is the so called SMC technique. SMC guarantees the robustness for dealing with parametric uncertainty and external disturbance (Edwards and Spurgeon, 1998; Perruquetti, 2002). SMC is a very popular tool for controlling nonlinear uncertain systems if the bounds on uncertainty are known (Slotine and Li, 1991; Utkin, 1977). But SMC suffers from the disadvantage of chattering which is the occurrence of high frequency finite amplitude oscillations. This chattering phenomenon occurs due to the use of discontinuous signum function and it could damage the actuators and systems. There have been many approaches for the removal of chattering like the use of approximation functions in place of the signum function, observer based SMC or HOSMC (Bartolini et al., 1998; Bartolini and Punta, 2000). One approach is to approximate the signum function in the control signal by a high gain saturation or sigmoid function and there by reducing the chattering (Utkin, 1992). However, although the chattering can be removed the robustness of the sliding mode is also compromised. Second method is based on the observer design which suppresses the high frequency oscillations of the control input (Lee and Utkin, 2007). A third approach is to design a HOSMC which retains the property of robustness, ensure finite time convergence and also reduces the chattering (Levant, 1993). But, in all of these approaches, it is assumed that the bounds on uncertainties are known.

If the bounds on uncertainties are unknown, then another approach known as ASMC can be used. In this approach, the gain of the controller is being tuned adaptively keeping in mind that the gain is not overestimated so that the magnitude of discontinuity in control is reduced to the minimum possible value for which sliding mode would exist (Bartolini and Punta, 2000; Plestan et al., 2010; Nasiri et al., 2014; Zhu and Khayati, 2017). But ASMC, like the conventional SMC, also suffers from the problem of chattering.

An adaptive sliding mode trajectory tracking controller is proposed to ensure asymptotic convergence of pose tracking errors and adaptive estimates in Lyapunov framework(Shtessel et al., 2016; Singh, 2016; Sun and Zheng, 2017; Fei and Lu, 2017). An adaptation scheme of the controller gain is derived via Lyapunov method to improve the control performance of the system (Levant, 2001a). The ASMC strategy for the stabilization of the system decreasing the chattering effect on the system due to the adaptive gain of the controller that makes it more robust and reliable in comparison with the state feedback control strategy. For this reason, this strategy is proved to be superior to the state feedback control approach, where the stability, small chattering and convergence of the sliding manifold in finite-time make it the best alternate for the control (Geng and Wang, 2016).

To overcome the chattering drawback, we have proposed a new AHOSMC algorithm in this paper. HOSMC uses the concept of not only keeping the sliding variable but also its derivatives to zero (Emel'yanov et al., 1996; Levant, 2003; Perruquetti, 2002; Levant, 2001b; Bartolini et al., 2003). r th order sliding mode is said to exist if the sliding variable and its $r-1$ derivatives are zero. In this paper, we have focused on the second order sliding mode for the systems having relative degree 1 with respect to the sliding variable and proposed an algorithm for the design of the first derivative of the control as the control rather than the actual control itself and then the gain for the same is chosen to vary adaptively according to the nearness to the sliding surface. For the systems having relative degree greater than 1, the extension is straightforward and has been mentioned. We have used adaptive sliding mode (Ying-Jeh Huang and Chang, 2008; Utkin and Poznyak, 2013; Plestan et al., 2010; Edwards and Shtessel, 2016; Nasiri et al., 2014; Liu et al., 2014; Durmaz et al., 2012) in which the gain of the controller is varied adaptively according to the nearness to the sliding surface so that the overestimation is avoided so that magnitude of discontinuity in control is reduced to the minimum possible value for which sliding mode would exist.

1.1 Main Contributions

The main contributions of this paper can be summarized as follows:

(1) An improved control algorithm is proposed, which uses the concept of AHOSMC.
(2) We have designed the first derivative of the control taking it as the virtual control rather than the actual control itself.
(3) The switching gain-adaptation law does not overestimate uncertainties.
(4) The proposed technique has been validated using numerical examples.

1.2 Organization of Paper

The remaining part of this paper is organized as follows. Section 2 mentions the problem which needs to be solved. In section 3, the concepts of AHOSMC has been mentioned briefly. After that, the control algorithm which has been designed using AHOSMC techniques is given. Section 4 shows the effectiveness of the given algorithm through simulation results. Finally, section 5 concludes the paper.

2. PROBLEM FORMULATION

Consider the nonlinear uncertain system

$$\dot{x}(t) = f(x(t)) + g(x(t))u \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}$ is the control input, $f : \mathbb{R}^n \to \mathbb{R}^n$ and $g : \mathbb{R}^n \to \mathbb{R}^n$ are smooth uncertain functions. Furthermore, it is assumed that $f(x)$ and $g(x)$ are bounded and $g(x) \neq 0 \forall x$. The bounds are unknown but they exist. The desired or the reference trajectory is given by $x_{ref}(t)$. Choose the sliding surface as

$$\sigma(x) = ce(t) \tag{2}$$

where $e(t) \in \mathbb{R}^n$ is the error vector $x(t) - x_{ref}(t)$ and $c \in \mathbb{R}^{1\times n}$ such that the polynomial $\sum_{i=1}^{n} c_i s^{i-1}$ is Hurwitz. Our aim is to design a control signal to steer $\sigma(x)$ to zero in finite-time. Furthermore, the chattering in the input should be removed.

Assuming that the relative degree of $\sigma(x)$ with respect to the control input u be 1 and considering the first and second time derivatives of $\sigma(x)$, we have

$$\begin{aligned} \dot{\sigma}(x) &= \phi(x) + \gamma(x)u \\ \ddot{\sigma}(x) &= \Phi(x,u) + \Gamma(x)\dot{u} \end{aligned} \tag{3}$$

If we are able to steer $\sigma(x)$ to zero in finite time by using a discontinuous control signal $\dot{u}$, then the corresponding u would be continuous and hence chattering could be avoided. Here, further we have to assume that all the functions $\phi(x), \gamma(x), \Phi(x)$ and $\Gamma(x)$ are bounded and also that $\gamma(x)$ and $\Gamma(x) \neq 0 \quad \forall x$.

3. DESIGN OF ADAPTIVE HIGHER ORDER SLIDING MODE CONTROLLER (AHOSMC)

The design procedure for the overall AHOSMC is carried out.

3.1 Higher Order Sliding Mode (HOSM)

Consider (1) but now the uncertainty bounds on $f(x)$ and $g(x)$ are supposed to be known.

The control objective in case of rth order sliding mode is to force the sliding variable σ and its $r-1$ derivatives to go to zero (Levant, 1993), i.e.

$$\sigma = \dot{\sigma} = \ddot{\sigma} = \cdots = \sigma^{r-1} = 0 \tag{4}$$

For the case of second order sliding, the control objective (4) becomes

$$\sigma = \dot{\sigma} = 0 \tag{5}$$

There are a number of algorithms to achieve the above objective. If the relative degree of $\sigma(x)$ with respect to the input is 1, then there are control design algorithms which consider designing of the virtual control v which would be the derivative of the actual control u. But these algorithms require the knowledge of bounds Φ_m, Φ_M and Γ_M described by

$$\Phi_m \leq \Phi(x,u) \leq \Phi_M, \quad 0 < \Gamma_m \leq \Gamma(x) \leq \Gamma_M \tag{6}$$

3.2 Proposed HOSMC

As mentioned previously, our solution algorithm would combine the features of both the ASMC and HOSMC. First of all, as mentioned previously, we will assume the relative degree with respect to the input be 1. Then without loss of generality, if we assume the system to be in phase variable form, then it could be represented as

$$\begin{aligned} \dot{x}_i &= x_{i+1} \qquad i = 1, 2, \cdots, n-1 \\ \dot{x}_n &= f(x) + g(x)u \end{aligned} \tag{7}$$

Choose the sliding surface as

$$\sigma = ce(t) = 0 \tag{8}$$

Assuming $y_1(x) = \sigma(x)$ and $y_2(x) = \dot{\sigma}(x)$, we have

$$\begin{aligned} \dot{y_1}(x) &= y_2(x) \\ \dot{y_2}(x) &= \Phi(x,u) + \Gamma(x)v \end{aligned} \tag{9}$$

Now, if we try to design v instead of u such that both $y_1(x)$ and $y_2(x)$ go to zero, then the establishment 2-sliding mode is guaranteed. Choose the manifold s for the above as

$$s = y_2 + |y_1|^{\frac{1}{2}}\text{sgn}(y_1) \tag{10}$$

Remark 1. (Singh, 2016) sgn denotes the standard signum function. The signum function exhibits the property that $s.\ sgn(s) = |s|$.

Proposition 1. If we are able to make s equal to zero, then both y_1 and y_2 would go to zero in finite time.

Proof 1. First lets consider the case when $y_1 > 0$. When s becomes zero, we have

$$y_2 = -y_1^{\frac{1}{2}} \tag{11}$$

which from (9) becomes

$$\dot{y}_1 = -y_1^{\frac{1}{2}} \tag{12}$$

Rearranging, we have

$$-\frac{dy_1}{y_1^{\frac{1}{2}}} = dt$$

Integrating both the sides under the limits $y_1(0)$ to 0 and 0 to t_f, we have

$$\int_{y_1(0)}^{0} -\frac{1}{y_1^{\frac{1}{2}}} dy_1 = \int_{0}^{t_f} dt$$

which gives the time in which y_1 becomes zero from its initial value $y_1(0)$ as $t_f = 2\sqrt{y_1(0)}$ which will always exist since $y_1(0)$ is positive by assumption.
Similar argument follows when y_1 is negative. This could also be proved by selecting proper Lyapunov function and then verifying the conditions of finite time convergence in the derivative of the function. Once, y_1 becomes zero, (11) guarantees that y_2 will also go to zero.

Auxiliary Problem Now, from proposition 1, we know that if s in (10) becomes zero, then we are sure about the establishment of 2-sliding mode. So, the auxiliary problem now is to design our control such that s goes to zero in finite time. From (10), we have

$$\dot{s} = \dot{y}_2 + \frac{1}{2}|y_1|^{-\frac{1}{2}}y_2$$

which from (9) becomes

$$\dot{s} = \Phi(x, u) + \Gamma(x)v + \frac{1}{2}|y_1|^{-\frac{1}{2}}y_2 \tag{13}$$

Now, v appears in the equation for $\dot{s}$, we can take take the Lyapunov function as s^2 and design v such that finite time convergence of s to zero is achieved. According to conventional SMC, we can design the control as

$$v = -\theta\text{sgn}(s) \tag{14}$$

where θ is a constant selected such that it is enough to counter the uncertainty bounds in all the terms and also gives a desired rate of convergence rate. But, now the uncertainty bounds are unknown. Therefore, we can choose θ based on the switching gain adaptive algorithm given below:

3.3 Adaptation Switching Gain Law

In the presence of time-varying uncertainty, this work proposes an adaptation law to update the value of the switching gain θ that confirms stability of the constrained system. The adaptation law chosen is given as

$$\dot{\theta} = \begin{cases} \bar{\theta}||s||\ sgn(||s|| - \epsilon) & if \quad \theta > \mu \\ \mu & if \quad \theta \leq \mu \end{cases} \tag{15}$$

where $\theta(0), \bar{\theta}, \mu$ and ϵ are positive controller parameters to be chosen. The value μ has been set to restrict the gain from going to non-positive value. A stronger condition to be chosen henceforth is $\theta(t) > \mu, \forall t > 0$. The parameter ϵ defines a region around the sliding surface that indicates closeness to the surface which will stop further variation in the switching gain as establishment of *ideal sliding mode* is not possible in real application.

Lemma 3.1. (Plestan et al., 2010) For a system defined in (1) with sliding variable (s) dynamics (13) controlled by the (14), and (15), the switching gain θ has an upper bound i.e., there exists a positive constant θ^* such that

$$\theta(t) \leq \theta^*, \forall t > 0.$$

Lemma. 3.1 has been utilized to ascertain the *finite-time convergence* of the system states to the sliding manifold.

Theorem 3.2. For the given uncertain system (1) with sliding variable (s) dynamics (13) controlled by (14) and (15), there exists a finite time $t_F > 0$ so that a real sliding mode is established $\forall t \geq t_F$.

Proof 2. Let a Lyapunov function candidate of the form

$$V = \frac{1}{2}s^2 + \frac{1}{2\gamma}(\theta - \theta^*)^2.$$

The derivative of the Lyapunov function candidate w.r.t time can be expressed as

$$\begin{aligned} \dot{V} &= s\dot{s} + \frac{1}{\gamma}(\theta - \theta^*)\dot{\theta} \\ &= s(\Phi(x, u) + \Gamma(x)v + \frac{1}{2}|y_1|^{-\frac{1}{2}}y_2) + \frac{1}{\gamma}(\theta - \theta^*)\dot{\theta} \\ &= s\big(\Phi(x, u) - \Gamma(x)\theta\ sgn(s) + \frac{1}{2}|y_1|^{-\frac{1}{2}}y_2\big) + \frac{1}{\gamma}(\theta - \theta^*)\dot{\theta} \end{aligned}$$

From (6) and Proposition 1, we can write

$$\dot{V} \leqslant (\Phi_M - \Gamma_m\theta^*)\ ||s|| + \frac{1}{\gamma}(\theta - \theta^*)\bar{\theta}\ ||s||\ sgn(||s|| - \epsilon)$$

Adding and subtracting a term $\theta^*\ ||s||$ we get

$$\begin{aligned} &= (\Phi_M - \Gamma_m\theta^*)\ ||s|| + (\theta - \theta^*)\big(-\Gamma_m||s|| \\ &\quad + \frac{\bar{\theta}}{\gamma}\ ||s||\ sgn(||s|| - \epsilon)\big) \end{aligned}$$

Introducing a positive value $\beta_k > 0$ as

$$\begin{aligned} &= (\Phi_M - \Gamma_m\theta^*)\ ||s|| - \beta_k\ ||\theta - \theta^*|| + (\theta - \theta^*)\big(-\Gamma_m||s|| \\ &\quad + \frac{\bar{\theta}}{\gamma}\ ||s||\ sgn(||s|| - \epsilon)\big) + \beta_k\ ||\theta - \theta^*|| \end{aligned}$$

From Lemma. 3.1, there always exist a θ^* such that $\theta - \theta^* < 0\ \forall\ t > 0$. This results in to

$$\begin{aligned} &= -(-\Phi_M + \Gamma_m\theta^*)\ ||s|| - \beta_k\ ||\theta - \theta^*|| - \big(-||s|| \\ &\quad + \frac{\bar{\theta}}{\gamma}\ ||s||\ sgn(||s|| - \epsilon) - \beta_k\big)\ ||\theta - \theta^*|| \\ &= -\beta_\sigma\ ||s|| - \beta_k\ ||\theta - \theta^*|| - \zeta \end{aligned}$$

where, $\beta_\sigma = (-\Phi_M + \Gamma_m\theta^*) > 0$ and $\zeta = \big(-\Gamma_m||s|| + \frac{\bar{\theta}}{\gamma}\ ||s||\ sgn(||s|| - \epsilon) - \beta_k\big)\ ||\theta - \theta^*||$.
Continuing

$$= -\beta_\sigma\sqrt{2}\frac{||s||}{\sqrt{2}} - \beta_k\sqrt{2\gamma}\frac{||\theta-\theta^*||}{\sqrt{2\gamma}} - \zeta$$
$$\leqslant -\min\{\beta_\sigma\sqrt{2}, \beta_k\sqrt{2\gamma}\}\left(\frac{||s||}{\sqrt{2}} + \frac{||\theta-\theta^*||}{\sqrt{2\gamma}}\right) - \zeta$$
$$\leqslant -\beta\ V^{1/2} - \zeta$$

where $\beta = \sqrt{2}\min\{\beta_\sigma, \beta_k\sqrt{\gamma}\}$
Case 1: if $||s|| > \epsilon$, ζ is positive if,

$$-\Gamma_m||s|| + \frac{\bar{\theta}}{\gamma}||\sigma|| - \beta_k > 0 \Rightarrow \gamma < \frac{\bar{\theta}\epsilon}{\Gamma_m\epsilon + \beta_k}$$

With this condition holding, the time derivative of the Lyapunov function becomes

$$\dot{V} \leqslant -\beta V^{1/2}$$

Case 2: if $||s|| \leqslant \epsilon$, ζ can be negative and it is not possible to prove negative-definiteness of $\dot{V}$.
This proves that $||s||$ converges to a region ϵ in finite time. But once inside ϵ, the trajectory of the sliding variable can not be ascertained. If at some t_{F_1} the sliding state goes beyond ϵ, i.e. $||s(t_{F_1})|| > \epsilon$, there exists another finite time t_{F_2} when the states will re-converge to the region ϵ. This proves the establishment of a real sliding mode.

Remark 2. The problem in the implementation of the above algorithm would be that $y_2(x)$ might be unavailable. In that case, we can use the first difference of $y_1(x)$ instead of $y_2(x)$.

Remark 3. If the relative degree of the sliding variable $\sigma(x)$ with respect to the control input is greater than 1, then the above algorithm could still be applied with a little modification. We just have to modify the manifold s in (10). The modification of the manifold should be done in a manner that $(r+1)-th$ sliding mode is established when s becomes zero.

4. SIMULATION RESULTS

We considered the following nonlinear system

$$\begin{aligned}
\dot{x}_1 =& x_2 \\
\dot{x}_2 =& x_3 \\
\dot{x}_3 =& -(1+0.3\sin(t))x_1^2 - (1.5+0.2\cos(t))x_2 \\
& -(1+0.4\sin(t))x_3 + (3+\cos(x_1))u
\end{aligned}$$

The aim is to make the system follow the desired trajectory given by $x_d = [\sin(t)\ \cos(t)\ \text{-}\sin(t)]^T$. We assume while designing the controller that the system dynamics are unknown. The sliding surface function is chosen to be (8) with c chosen as $[12 \quad 5 \quad 7]$. Clearly, the relative degree with respect to the input is 1. The value of ϵ is taken to be 0.001. The simulation results are as shown:

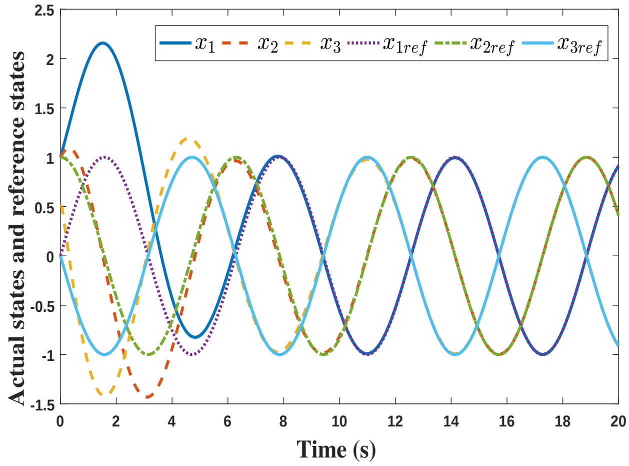

Fig. 1. Controlled states using the proposed method

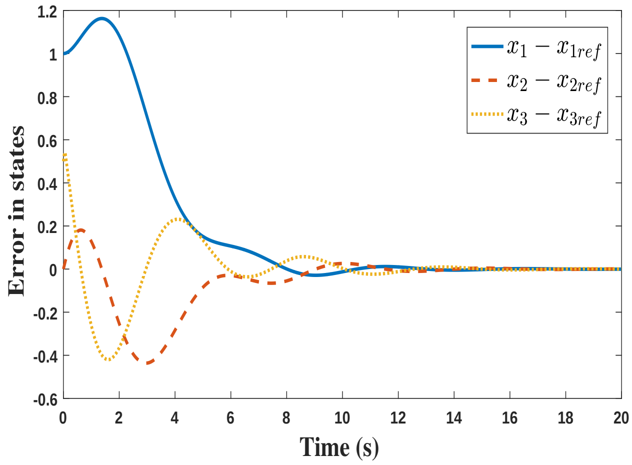

Fig. 2. Error in the states

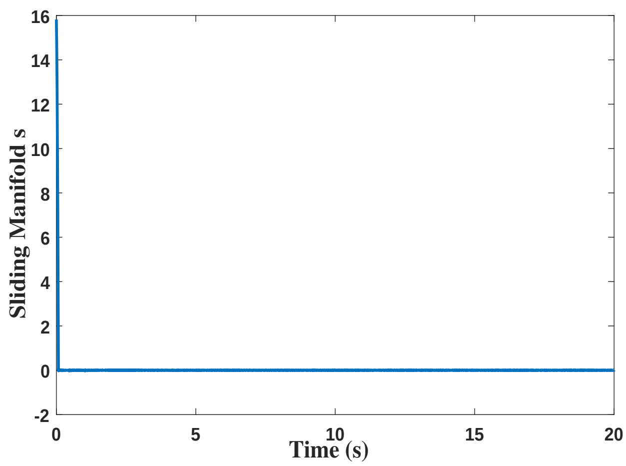

Fig. 3. Manifold s

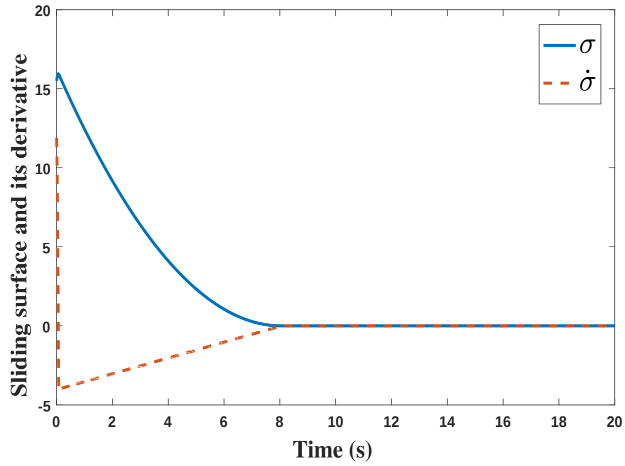

Fig. 4. Sliding surface (σ) and its derivative ($\dot{\sigma}$)

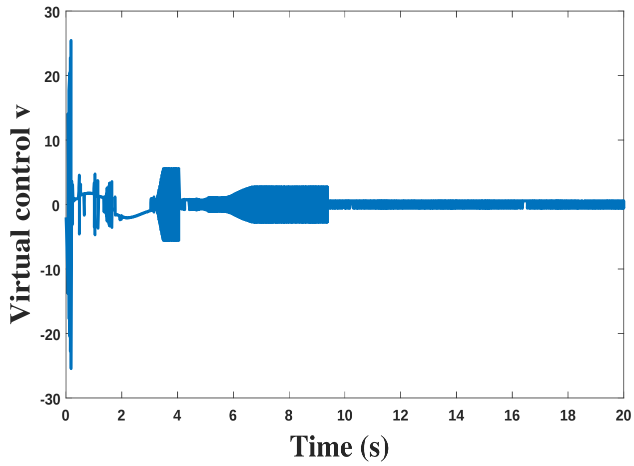

Fig. 5. Virtual control v

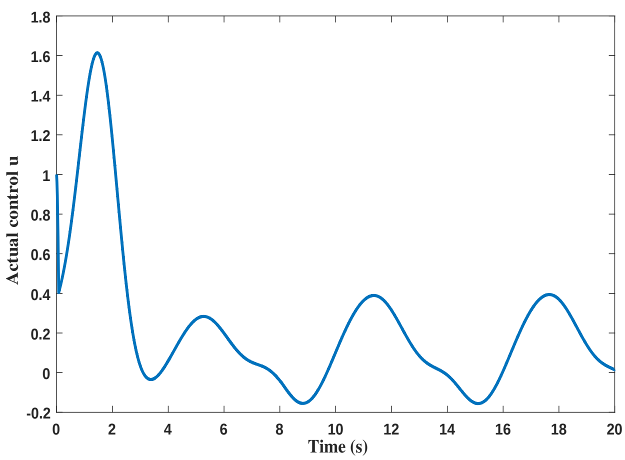

Fig. 6. Chattering free actual control u

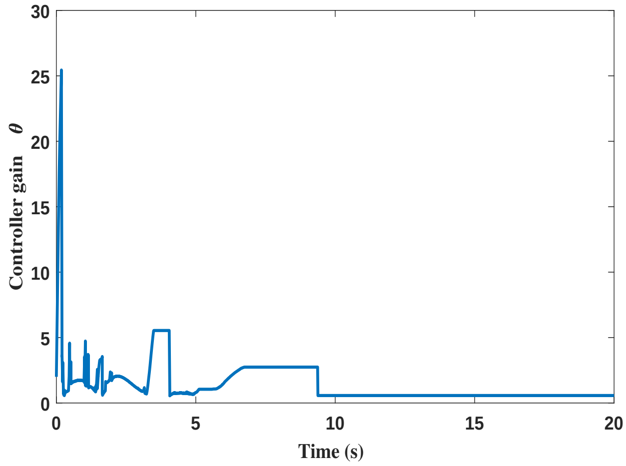

Fig. 7. Adaptive controller gain θ

Proposed AHOSMC state trajectories response are shown in Fig. 1. Error between the actual and reference states responses are shown in Fig. 2. The sliding manifold s obtained using the AHOSMC is smooth as shown in Fig. 3. The sliding surface σ and its derivative responses are shown in Fig. 4. Virtual control input v response is given in Fig. 5. Smooth chattering free control input u is shown in Fig. 6. The convergence of the adaptive gain θ is confirmed as illustrated in Fig. 7. From the simulation results, it is clear that there is no chattering in the control u while the desired performance is achieved.

5. CONCLUSIONS

This paper proposes an effective higher-order based adaptive sliding mode algorithm for the control of unknown bounded systems. The control which is designed using the algorithm given in the paper is found to be free from chattering. The controller gain is varied adaptively and overestimation of the gain is also avoided through the algorithm mentioned. As a precaution, to avoid the increase of controller gain boundlessly, the concept of real sliding is used. Simulation results illustrated the benefits of the our proposed AHOSMC over the existing ASMC method.

The application of the proposed concept in the various fields, such as sliding mode control (Singh and Sharma, 2012; **?**; Singh and Janardhanan, 2015), Stochastic systems (Singh and Janardhanan, 2017a; Sharma et al., 2017; Singh and Janardhanan, 2017b) are left for the future.

REFERENCES

Bartolini, G., Ferrara, A., and Usani, E. (1998). Chattering avoidance by second-order sliding mode control. *Automatic Control, IEEE Transactions on*, 43(2), 241–246.

Bartolini, G., Pisano, A., Punta, E., and Usai, E. (2003). A survey of applications of second-order sliding mode control to mechanical systems. *International Journal of Control*, 76(9-10), 875–892.

Bartolini, G. and Punta, E. (2000). Chattering elimination with second-order sliding modes robust to coulomb friction. *J. Dyn. Sys., Meas., Control 122, 679-686*

(2000) (8 pages); doi: 10.1115/1.1316797, 122(4), 679–686.

Durmaz, B., zgren, M.K., and Salamci, M.U. (2012). Sliding mode control for non-linear systems with adaptive sliding surfaces. *Transactions of the Institute of Measurement and Control*, 34(1), 56–90.

Edwards, C. and Spurgeon, S. (1998). *Sliding Mode Control: Theory And Applications*. Series in Systems and Control. Taylor & Francis.

Edwards, C. and Shtessel, Y.B. (2016). Adaptive continuous higher order sliding mode control. *Automatica*, 65, 183 – 190. URL http://www.sciencedirect.com/science/article/pii/S0005109815005130.

Emel'yanov, S., Korovin, S., and Levant, A. (1996). High-order sliding modes in control systems. *Computational Mathematics and Modeling*, 7(3), 294–318.

Fei, J. and Lu, C. (2017). Adaptive sliding mode control of dynamic systems using double loop recurrent neural network structure. *IEEE Transactions on Neural Networks and Learning Systems*, PP(99), 1–12. doi:10.1109/TNNLS.2017.2672998.

Geng, J. and Wang, Y. (2016). A finite-time sliding-mode controller design for convergence at a desired time and global robust control. In *2016 35th Chinese Control Conference (CCC)*, 550–555. doi:10.1109/ChiCC.2016.7553142.

Lee, H. and Utkin, V.I. (2007). Chattering suppression methods in sliding mode control systems. *Annual Reviews in Control*, 31(2), 179 – 188.

Levant, A. (2001a). Universal single-input-single-output (siso) sliding-mode controllers with finite-time convergence. *IEEE Transactions on Automatic Control*, 46(9), 1447–1451. doi:10.1109/9.948475.

Levant, A. (1993). Sliding order and sliding accuracy in sliding mode control. *International Journal of Control*, 58(6), 1247–1263.

Levant, A. (2001b). Universal single-input-single-output (siso) sliding-mode controllers with finite-time convergence. *Automatic Control, IEEE Transactions on*, 46(9), 1447–1451.

Levant, A. (2003). Higher-order sliding modes, differentiation and output-feedback control. *International Journal of Control*, 76(9-10), 924–941.

Liu, Z., Su, H., and Pan, S. (2014). A new adaptive sliding mode control of uncertain nonlinear systems. *Asian Journal of Control*, 16(1), 198–208.

Nasiri, A., Nguang, S.K., and Swain, A. (2014). Adaptive sliding mode control for a class of mimo nonlinear systems with uncertainties. *Journal of the Franklin Institute*, 351(4), 2048 – 2061. Special Issue on 2010-2012 Advances in Variable Structure Systems and Sliding Mode Algorithms.

Perruquetti, W. (2002). *Sliding Mode Control in Engineering*. Marcel Dekker, Inc., New York, NY, USA.

Plestan, F., Shtessel, Y., Brgeault, V., and Poznyak, A. (2010). New methodologies for adaptive sliding mode control. *International Journal of Control*, 83(9), 1907–1919.

Sharma, N.K., Singh, S., Janardhanan, S., and Patil, D.U. (2017). Stochastic higher order sliding mode control. In *25th Mediterranean Conference on Control and Automation*, 649–654.

Shtessel, Y., Fridman, L., and Plestan, F. (2016). Adaptive sliding mode control and observation. *International Journal of Control*, 89(9), 1743–1746.

Singh, S. and Janardhanan, S. (2015). Stochastic variable structure control of discrete time LTI systems with bounded disturbances. In *2015 International Workshop on Recent Advances in Sliding Modes (RASM)*, 1–6.

Singh, S. and Janardhanan, S. (2017a). Functional observer design for linear discrete-time stochastic system. In *2017 Australian and New Zealand Control Conference (ANZCC)*, 175–178.

Singh, S. (2016). Single input sliding mode control for hyperchaotic lu system with parameter uncertainty. *International Journal of Dynamics and Control*, 4(4), 504–514.

Singh, S. and Janardhanan, S. (2017b). Sliding mode control-based linear functional observers for discrete-time stochastic systems. *International Journal of Systems Science*, 48(15), 3246–3253.

Singh, S. and Sharma, B.B. (2012). Sliding mode control-based stabilisation and secure communication scheme for hyperchaotic systems. *International Journal of Automation and Control*, 6(1), 1–20.

Slotine, J.J.E. and Li, W. (1991). Applied nonlinear control. *NJ: Prantice-Hall, Englewood Cliffs*.

Sun, L. and Zheng, Z. (2017). Adaptive sliding mode control of cooperative spacecraft rendezvous with coupled uncertain dynamics. *American Institute of Aeronautics and Astronautics*, 54(3), 652–661.

Utkin, V. (1977). Variable structure systems with sliding modes. *Automatic Control, IEEE Transactions on*, 22(2), 212–222.

Utkin, V.I. and Poznyak, A.S. (2013). Adaptive sliding mode control with application to super-twist algorithm: Equivalent control method. *Automatica*, 49, 39–47.

Utkin, V. (1992). *Sliding Modes in Control and Optimization*. Communication and control engineering series. Springer-Verlag.

Ying-Jeh Huang, T.C.K. and Chang, S.H. (2008). Adaptive sliding-mode control for nonlinear systems with uncertain parameters. *IEEE Transactions on Systems, Man, and Cybernetics - Part B: Cybernetics*, 38, 534–539.

Zhu, J. and Khayati, K. (2017). On a new adaptive sliding mode control for mimo nonlinear systems with uncertainties of unknown bounds. *International Journal of Robust and Nonlinear Control*, 27(6), 942–962.