Participation of Microgrids in Frequency Regulation Markets

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Abstract— **The increasing ubiquity of distributed energy resources (DERs) in the power grid provides an opportunity for them to collectively participate in frequency regulation. In this paper, we propose a framework for the participation of microgrids, each acting as an aggregator of certain number of DERs, in the frequency regulation market when operating in grid-connected mode. Our treatment covers both the determination of bids for the market clearance stage and mechanisms for the real-time allocation of the regulation signal. Regarding market clearance, we develop abstractions of the microgrid capabilities regarding capacity, cost of generation, and ramp rates as a function of the individual energy resources that comprise it. Regarding real-time allocation of the regulation signal, we formulate an optimization problem that seeks to minimize the collective cost of the microgrids while respecting operational constraints. Since this problem is not always feasible due to the ramp rate constraints of the microgrids, we provide an alternative formulation that incorporates the minimization of the deviation between the procured collective regulation and the required one. We synthesize an algorithm whose execution involves the coordination of the system operator and the microgrids, and establish its asymptotic convergence to the desired optimizers. Simulations illustrate our results.**

I. Introduction

Recently, there is an increasing interest in integrating distributed energy resources (DERs) into the grid in general and for frequency regulation in particular. Since individual DERs do not have enough capability to participate in the market on their own, the vision is to integrate them through aggregators or Distributed Energy Resources Providers (DERPs) that act as virtual power plants: they do not own generation but instead coordinate response of distributed energy resources. This architecture has recently being proposed by the California ISO (CAISO) to offer aggregators of DERs the opportunity to sell into its marketplace [1]. However, such integration raises a number of challenges including, but not limited to, the determination of capacity bounds and cost of generation functions for such aggregators, the coordination among different aggregators to service a given regulation signal, and the design of mechanisms for the coordination of individual resources under the control of an aggregator.

Literature Review: The U.S. Federal Energy Regulatory Commission (FERC) issued Order 755 [2] requiring RTOs to compensate energy resources based on the actual services provided. The payment to resources comprises of two parts, the capacity and performance payments. The capacity payment compensates resources for their provision of regulation capacity. The performance payment reflects the accuracy of the tracking of the allocated regulation signal. [3] describes how the different RTOs across the United States have implemented FERC Order 755 for participation of individual resources in frequency regulation market. In the literature on power networks and smart grid, some works have considered the possibility of obtaining frequency regulation services from collections of homogeneous loads such as electric vehicles (EVs) and thermostatically controlled loads (TCLs), cf. [4], [5]. The work [6] presents a method to model flexible loads as a virtual battery for providing frequency regulation. [7] proposes the use of aggregators to integrate heterogeneous loads such as heat pumps, supermarket refrigerators and batteries present in industrial buildings to provide frequency regulation. The work [8] describes the challenges that need to be overcome for providing frequency regulation by DERs for some European countries. The work [9] provides a framework to emulate virtual power plants (VPPs) via aggregations of DERs and provide regulation services taking into account the power flow constraints. [10] provides a dispatch strategy for an aggregate of ON/OFF devices to provide frequency regulation. However, the approaches in [9], [10] assume that the allocated signal from the RTO is available to the aggregator. In the context of microgrids, work has also been done [11], [12] in the design of mechanisms for optimally allocating a given signal among the DERs within the microgrid. [13] applies machine learning to forecast the power capacity of VPPs. The work [14] provides a framework for optimal bidding and dispatch of multiple VPPs. Here, we focus on (i) enabling the participation of microgrids in frequency regulation markets operated by the RTO through the identification of appropriate bids and (ii) the coordination among RTO and microgrids to efficiently dis-aggregate the regulation signal.

Statement of Contributions: We propose a framework for the participation of microgrids in the frequency regulation market. We start with a description of the current stages of this market consisting of market clearance, allocation from the RTO to the participating energy resources of the regulation signal, and finally the actual real-time tracking of the regulation signal. The identification of the limitation of current practice sets the stage for the paper contributions. Our first contribution is the identification of abstractions for the capacity, cost of generation, and ramp rates of a microgrid as a combination of the individual energy resources that compose it, along with formal description of some of its smoothness and convexity properties. Equipped with these abstractions, a microgrid can submit bids to participate in the market clearance stage. Our second contribution is the design of an algorithmic solution to the RTO-DERP coordination problem to dis-aggregate the regulation signal taking into

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account both cost of generation and operational limits. The proposed algorithm involves communication between the RTO and the aggregators in its execution and is guaranteed to asymptotic converge to the desired optimizers. For reasons of space, all proofs are omitted and will appear elsewhere.

II. Preliminaries

In this section, we present our notational conventions and review some basic concepts.

Notation: Let $\mathbb{R}$ and $\mathbb{Z}$ be the set of real numbers and integers, respectively. We let $|X|$ and $\mathrm{co}(X)$ denote the cardinality and convex hull of a set X, respectively. We use $[x]^+$ to denote $\max\{x, 0\}$ and $[x]^+_a$ to denote $[x]^+$ if $a > 0$ and 0 if $a \leq 0$. For a real-valued function $L : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}$, we denote by $\nabla_x L$ the partial derivative of L with respect to its argument x.

Graph theory: We denote a directed graph by $G = (\mathcal{V}, E)$, with $\mathcal{V}$ as the set of vertices and $E \subseteq \mathcal{V} \times \mathcal{V}$ as the set of edges. The *incidence matrix* M of a directed graph is a $|\mathcal{V}| \times |E|$ matrix such that $M_{i,j} = 1$ if the edge e_j leaves vertex v_i, -1 if it enters vertex v_i, and $M_{i,j} = 0$ otherwise. Due to this structure of M, every column of M has only two non-zero entries. Note that, $\mathbf{1}^T M = 0$, where $\mathbf{1}$ is the vector of 1s of appropriate dimension.

Convex Analysis: A function $L : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}$ is *globally convex-concave* if for any $(\overline{x}, \overline{z}) \in \mathbb{R}^n \times \mathbb{R}^m$, the functions $x \mapsto L(x, \overline{z})$ and $z \mapsto L(\overline{x}, z)$ are convex and concave, respectively. A point $(x^*, z^*) \in \mathbb{R}^n \times \mathbb{R}^m$ is a *global min-max saddle-point* of $L : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}$ if

$$L(x^*, z) \leq L(x^*, z^*) \leq L(x, z^*) \quad \forall (x, z) \in \mathbb{R}^n \times \mathbb{R}^m \quad (1)$$

In this paper, we focus only on the global min-max saddle points, which are just a particular case of saddle points and refer to them as saddle points throughout the paper for the sake of simplicity. We use $\mathrm{Saddle}(L)$ to denote the set of saddle points of L. At each of the saddle points, $\nabla_x L(x^*, z^*) = \nabla_z L(x^*, z^*) = 0$.

III. Frequency Regulation with Microgrids

We are interested in solving the problem of how to coordinate the actions of aggregators to participate in frequency regulation. An aggregator is an entity that aggregates the actions of a group of distributed energy resources to act as a single, virtual whole. Here, we identify an aggregator with a microgrid, but in general it could correspond to other scenarios (such as, for instance, a collection of microgrids).

A. Current Practice and Limitations

The frequency regulation market is operated by the RTO with the purpose of restoring power balance to the grid. The RTO coordinates the response of participating energy resources in a centralized fashion to assign the regulation signal. The procedure has the following stages, see e.g., [3]:

[CP1]: Market clearance. All participating resources submit capacity bids, capacity price bids, and mileage price bids to the RTO. Capacity bids are the maximum amount of regulation up (or down) that the resource can provide. Capacity price bids give the cost of providing these regulations. Mileage is the sum of the absolute change in AGC set points. The mileage price bid is the cost for unit change in regulation. Typically, resources do not submit any mileage bids and, instead, expected mileages are calculated on the basis of historical data. The RTO clears the market with a uniform scheme for capacity and mileage, and sends to each resource its capacity and mileage allocation. This process only happens once per regulation period.

[CP2]: Allocation of regulation signal to each resource. The RTO sends the regulation set points to each of the procured energy resources every 2-4 seconds for the entire regulation period, which is usually 10-15 minutes. The regulation set points are calculated from the AGC signal in real time in proportion to the procured mileage of each resource. In case the assigned capacity of a resource is violated, the overshoot power is redistributed to the other resources in proportion of their assigned mileages.

[CP3]: Real-time tracking of regulation signal. After the RTO allocates the regulation signal to each resource, the actual real-time tracking of the signal has to be done by each resource.

The current centralized way of carrying out signal allotment from the RTO to the resources in [CP2] relies on a fixed number of resources with fixed generation capacities that are available for the entire regulation period. Furthermore, current allocation schemes do not take into account the operational costs of the resources, which in turn results in non-optimal power allocation. This is even more problematic in the context of microgrid participation, as microgrids themselves are subject to variabilities and uncertainties associated with the DERs inside them. Instead, we argue that the assignment of the regulation signal could be done, at each time step, in a way that optimizes the aggregate cost functions of the resources and takes into account their (possibly dynamic) operational limits. We refer to this approach as the RTO-DERP coordination problem. This idea has also been pointed out in the past by CAISO even for traditional energy resources, cf. [15].

B. Problem Statement

Consider m microgrids, each controlled by an aggregator, participating in the frequency regulation market. To carry out the submission of bids in [CP1], each aggregator needs to quantify the maximum up/down regulation capacity that the microgrid can provide, the cost of providing such regulation, and the guaranteed ramp rate at which the microgrid can change its power contribution. This is specially important as microgrids themselves are a combination of energy resources whose availability changes over time and whose performance might be substantially different at different regulation periods. Our first goal is therefore to provide meaningful abstractions for these objects.

The RTO-DERP coordination problem advocated in [CP2] consists of an economic dispatch problem with ramp rate constraints *at every instant of the regulation interval*. Formally, for ΔP_R regulation at a given time instant, we have

$$\begin{aligned} \min_{\Delta P} \quad & f(\Delta P) = \sum_{\alpha=1}^{m} f_\alpha(\Delta P_\alpha) \qquad (2) \\ \text{s.t.} \quad & \sum_{\alpha=1}^{m} \Delta P_\alpha = \Delta P_R \\ & \underline{\Delta P_\alpha} \leq \Delta P_\alpha \leq \overline{\Delta P_\alpha} \quad \forall \alpha = 1, \cdots, m \\ & |\Delta P_\alpha - \Delta P_\alpha^-| \leq R_\alpha(\Delta P_\alpha^-) \quad \forall \alpha = 1, \cdots, m \end{aligned}$$

where $\Delta P \in \mathbb{R}^m$ is the vector of regulation power from the microgrids, $f_\alpha(\Delta P_\alpha)$ is the cost of ΔP_α regulation for microgrid α, $\underline{\Delta P_\alpha}$ and $\overline{\Delta P_\alpha}$ are the lower and upper bounds of regulation for microgrid α determined as output of [CP1], ΔP_α^- is the regulation that the microgrid α was providing at the previous instant, and $R_\alpha(\Delta P_\alpha)$ is the ramp rate of the microgrid when it is providing regulation ΔP_α. Note that this problem requires the identification of cost and ramp rate functions for the microgrids described in our first goal above. Also note that, because of the ramp constraints present in (2), this problem might not be feasible always (since mileage requirements set by the RTO for participation capture the average mileage required, and not extreme cases). Our second goal is therefore to address this problem in a way that minimizes the error between the procured collective regulation and the required one, and synthesize schemes that allow the aggregators to coordinate their response to do so.

IV. Microgrid Capacity, Cost, and Ramp Rate

Consider a microgrid with $n \in \mathbb{Z}_{>1}$ buses whose interconnection is described by $G = (\mathcal{N}, E)$. We partition the set of nodes as $\mathcal{N} = \mathcal{N}_g \cup \mathcal{N}_l \cup \{1\}$, where $\mathcal{N}_g$ is the set of controllable nodes, $\mathcal{N}_l$ is the set of uncontrollable nodes (or loads), and without loss of generality, node 1 connects to the bulk power grid. We assume that the network, inverter filter, and voltage controller dynamics are fast enough so that we can model them as power injections with no dynamics [16]. We adopt the convention that the value of the power injection is negative if it consumes power and vice versa. The power level for each controllable node $p \in \mathcal{N}_g$ is denoted by g_p, with g_p^0 denoting the baseline generation/consumption. The value of each uncontrollable node $q \in \mathcal{N}_l$ is denoted by l_q. Inside the microgrid, the total load combined with the transmission losses must be equal to the total generation combined with the incoming power through the tie-line, i.e.,

$$\sum_{p \in \mathcal{N}_g} g_p + P = \sum_{q \in \mathcal{N}_l} l_q + \text{losses}, \qquad (3)$$

where P is the incoming power through the tie-line. The baseline value of this power is P^0. When the microgrid is not providing any regulation (i.e., the incoming baseline power remains constant), the power level of uncontrollable nodes inside the microgrid could change during the regulation period and, as a result, the power level of the controllable loads has to be adjusted so that (3) is satisfied with $P = P^0$. When the microgrid provides frequency regulation service, the value of the tie-line power P is instead $P^0 + \Delta p$, where Δp is the allocated AGC signal. In addition to constraint (3) on the net power balance, we have the power flow equations that need to be satisfied. Let $V_i (= |V_i| e^{j\theta_i})$ be the phasor voltage at the $i-$th bus with $|V_i|$ as the magnitude and θ_i as the phase angle. We make the following assumptions and then introduce the power flow equations.

Assumption 1: (Approximate constant voltage). The voltage magnitude of every the bus is approximately 1 p.u.

Assumption 2: (Small angle difference between connected buses). The angle difference of every $\{i, k\} \in E$ is bounded by ϕ as $|\theta_i - \theta_k| \leq \phi$. Furthermore, ϕ is sufficiently small so that $\cos(\theta_i - \theta_k) \approx 1$.

With Assumptions 1 and 2, power flow from bus i to bus k is given by $f_{ik} = B_{ik} \sin(\theta_i - \theta_k) + G_{ik}$, where G_{ik} and B_{ik} are the conductances and the susceptances between bus i and k. The power injection $\mathcal{P}_i$ at node i is given by

$$\mathcal{P}_i = \sum_{k=1}^{n} f_{ik} = \sum_{k=1}^{n} B_{ik} \text{sin}(\theta_i - \theta_k) + C_i,$$

where $C_i = \sum_{k=1}^{n} G_{ik}$. For the whole microgrid, we have

$$\begin{bmatrix} P & g^\top & -l^\top \end{bmatrix}^\top = MB \text{sin}(M^T \theta) + C \qquad (4)$$

where $C \in \mathbb{R}^{|\mathcal{N}|}$, $P \in \mathbb{R}$ is the incoming tie-line power, $g \in \mathbb{R}^{|\mathcal{N}_g|}$ and $l \in \mathbb{R}^{|\mathcal{N}_l|}$ are the vectors of controllable and uncontrollable nodes (which we take as constant for now), respectively, $M \in \mathbb{R}^{|\mathcal{N}| \times |E|}$ is the incidence matrix for arbitrary edge orientation, $B \in \mathbb{R}^{|E| \times |E|}$ is the diagonal matrix of absolute line susceptances and $\theta \in \mathbb{R}^{|\mathcal{N}|}$ is the vector of phase angles. Note that, due to the structure of M, net power balance constraint (3) is satisfied as long as (4) holds. Since (4) remains problematic for analysis, we make the following assumption to further simplify it.

Assumption 3: G is a graph with non-overlapping loops. Following [17], we next rewrite (4) with Assumption 3

$$\begin{aligned} \begin{bmatrix} P & g^\top & -l^\top \end{bmatrix}^\top &= Mf + C \qquad (5a) \\ |f| &\leq \overline{f} \qquad (5b) \end{aligned}$$

where, $f \in \mathbb{R}^{|E|}$ is the vector of line flows with $\overline{f} \in \mathbb{R}^{|E|}$ as the vector of maximum permissible flows. Note that this equation is affine due to the presence of C. One can always find an invertible mapping from f to some $\hat{f}$ such that $\hat{f} = Mf + C$, and express (5b) in terms of $\hat{f}$. Using this fact, we drop C throughout the rest of the paper for simplicity.

A. Capacity Bounds

A microgrid can find the maximal capacity for up (or down) regulation by solving an optimization problem. We show how the optimization is posed for up-regulation as the one for down regulation is very similar. For up-regulation, the power consumption of the microgrid is less than the baseline power. Since the baseline power is constant for the regulation period, this is equivalent to minimizing P while satisfying the power flow constraints. Formally, the problem is

$$\begin{aligned} \min_{g,f} \quad & P \\ \text{s. t.} \quad & \underline{g_p} \leq g_p \leq \overline{g_p} \quad \forall p \in \mathcal{N}_g \qquad (6) \\ & \begin{bmatrix} P & g^\top & -l^\top \end{bmatrix}^\top = Mf, \quad |f| \leq \overline{f} \end{aligned}$$

where, $\underline{g_p}$ and $\overline{g_p}$ are the minimum and maximum possible power levels of the $p-$th controllable node. If $\underline{P}$ denotes the solution of (6), then the maximum up regulation is $\overline{c} = \underline{P} - P^0$. Similarly, the maximum down regulation $\underline{c}$ is obtained by solving the corresponding maximization problem.

B. Ramp Rate Function

We focus on the ramp up rate (the discussion for ramp down rate is analogous). The ramp rate R_m of the microgrid depends on the operating point of the controllable nodes (which is related to the flows by (5)). Formally, $R_m(g)$ is

$$\begin{aligned}
\max_{\Delta g, \Delta f} \quad & \mathbf{1}^T \Delta g \\
\text{s.t.} \quad & \Delta g \leq R \qquad\qquad (7) \\
& \left[(P - \mathbf{1}^T \Delta g) \quad (g + \Delta g)^\top \ -l^\top \right]^\top = M(f + \Delta f) \\
& |f + \Delta f| \leq \overline{f}
\end{aligned}$$

Here, R is a $|\mathcal{N}_g|$-vector whose component R_p is the nominal ramping capacity of the controllable node p and f is the vector of line flows corresponding to the operating point g. Clearly, if there were no constraints on the power flows, then the ramping capacity would be $\mathbf{1}^T R$. In general, however, the presence of flow constraints may prevent every controllable node from ramping at its full capacity.

For a given regulation power Δp, there could be more than one feasible operating point. As a result, the ramp rate as a function of regulation power is not uniquely defined. We address the issue by defining

$$R_m(\Delta p) = \max_{g^* \in S(\Delta p)} R_m(g^*),$$

where $S(\Delta p)$ corresponds to the minimizers of the cost of producing the said amount of regulation respecting the power flow and capacity constraints. If the cost functions for all the controllable nodes are assumed to be convex, g^* is a decreasing function with respect to Δp, which means that at least one component of g^* would decrease with increase in Δp (using the convention that up regulation is negative). Using this fact, we conclude that R_m as a function of Δp is non-decreasing, with maximum possible value as $\mathbf{1}^T R$.

C. Cost Function

Each aggregator should calculate the cost of providing certain amount of regulation by capturing the effect of operating the controllable nodes away from their baseline operating points. At operating point g, the cost for the aggregator is

$$h(g) = \sum_{p \in \mathcal{N}_g} h_p(g_p), \qquad (8)$$

where h_p corresponds to the cost of operating node p away from its baseline level g_p^0. The total regulation that the aggregator provides is the combination of individual regulations of controllable loads. Therefore, for a specified regulation level Δp, one would choose the value of g that minimizes (8) while satisfying the power flow constraints (5) and the capacity constraints on each controllable node. However, in addition to the power flow and capacity constraints, the ramp rate constraints might limit the movement from optimal point at one time instant to the next. This motivates us to define cost as a function of the operating ramp rate r_m too. Formally, $\mathbf{f}(\Delta p, r_m)$ is the result of

$$\begin{aligned}
\min_{g, \Delta g, f, \Delta f} \quad & h(g + \Delta g) \\
\text{s.t.} \quad & \underline{g} \leq g + \Delta g \leq \overline{g}, \quad \Delta g \leq R, \\
& \left[(P^0 + \Delta p) \quad (g + \Delta g)^\top \ -l^\top \right]^\top = M(f + \Delta f), \\
& |f + \Delta f| \leq \overline{f}, \qquad\qquad (9) \\
& \underline{g} \leq g \leq \overline{g}, \quad |f| \leq \overline{f}, \\
& \left[(P^0 + \Delta p + r_m) \quad g^\top \ -l^\top \right]^\top = Mf,
\end{aligned}$$

where, $(g + \Delta g, f + \Delta f)$ and (g, f) are the vectors of the power levels of controllable nodes and line flows when the microgrid is providing regulation Δp and $\Delta p + r_m$, respectively. We have the capacity constraints for the individual controllable nodes and the flow limit constraints for both values of regulation power and the ramp constraints in transitioning from $\Delta p + r_m$ to Δp. This cost function would be used in the optimal allocation of the regulation signal to the microgrids. The next result describes the convexity properties of the cost function.

Lemma 4.1: (Convexity of the cost function). If h is (strictly) convex, then $\mathbf{f}$ is (strictly) convex.

Since it is desired to solve the RTO-DERP problem in a short span of time, we propose that the cost functions of the microgrids remain a function of only the regulation power. This cost function, say f, would be calculated by evaluating (9) at $r = \min_{\Delta p \in [\overline{c}, \underline{c}]} R_m(\Delta p)$ (the minimum value of the ramp rate), i.e.,

$$f(\Delta p) = \mathbf{f}(\Delta p, r)$$

With this definition of cost at the minimum ramp rate, we ensure that the cost is always finite.

Remark 4.2: (Time-varying loads). Along the regulation period, the power levels of all the uncontrollable nodes might change randomly with their maximum and minimum values given by $\underline{l_q}$ and $\overline{l_q}$, respectively ($\forall q$). Since the power levels of the controllable nodes needs to be adjusted for this variation, we can view the baseline power of controllable nodes as a function of l. The functions h used to calculate the microgrid cost function depend upon the difference $(g_p - g_p^0)$ for all p and not on the absolute values of g_p. Therefore, the construction of the cost function remains similar. For the regulation bounds, the maximum up (minimum down) regulation that the microgrid could provide would reduce and we calculate it using the vector of maximum (minimum) power levels of uncontrollable nodes. •

D. Bids for Participation in Market Clearance

Here we specify the bid information used by each aggregator to participate in [CP1]. The capacity bids are the regulation bounds obtained in Section IV-A. The capacity price bids are given by the value of $f(\Delta p)/\Delta p$ at the regulation bounds, i.e., $f(\overline{c})/\overline{c}$ and $f(\underline{c})/\underline{c}$. Regarding mileage, the aggregator bids with the mileage proportional to r (calculated as the product of the regulation period and r), as that is the guaranteed ramp rate that the microgrid would always be able to provide. This might result in two mileage bids for RTOs (e.g., CAISO) that treat up and down regulation as different products.

V. RTO-DERP Coordination Problem

Here we describe our proposed algorithmic solution for the RTO-DERP coordination problem in [CP2] to disaggregate the regulation signal. Equipped with the microgrids' capacities and cost and ramp rate functions identified in Section IV, the aggregators seek to solve, at each instant of the regulation period, the optimization problem (2). However, as noted above, this problem might not always be feasible due to the ramp constraints. Our first step is then to reformulate the optimization problem in a way that lends itself to the identification of solutions that minimize the error between the procured regulation and the required regulation whenever (2) is not feasible. Without loss of generality, considering the case of positive regulation, we can reformulate (2) as

$$\begin{aligned} \min_{\Delta P} \quad & f(\Delta P) \\ \text{s.t.} \quad & \Delta P_R \leq \sum_{\alpha=1}^{m} \Delta P_\alpha \\ & \underline{\Delta P_\alpha} \leq \Delta P_\alpha \leq \overline{\Delta P_\alpha} \quad \forall \alpha \\ & |\Delta P_\alpha - \Delta P_\alpha^-| \leq R_\alpha(\Delta P_\alpha^-) \quad \forall \alpha \end{aligned} \tag{10}$$

Following [18], consider the non-smooth penalty function

$$f^\epsilon(\Delta P) = f(\Delta P) + \frac{1}{\epsilon}[\Delta P_R - \sum_{\alpha=1}^{m} \Delta P_\alpha]^+$$

with $\epsilon > 0$ and define the problem

$$\begin{aligned} \min_{\Delta P} \quad & f^\epsilon(\Delta P) \\ \text{subject to} \quad & \underline{\Delta P_\alpha} \leq \Delta P_\alpha \leq \overline{\Delta P_\alpha} \quad \forall \alpha \\ & |\Delta P_\alpha - \Delta P_\alpha^-| \leq R_\alpha(\Delta P_\alpha^-) \quad \forall \alpha \end{aligned} \tag{11}$$

Note that if f is convex, then so is f^ϵ, and hence (11) is convex. According to [18], if (10) is convex, has a non-empty and compact solution set and satisfies the refined Slater condition, then (10) and (11) have exactly the same solutions if

$$\frac{1}{\epsilon} > ||\lambda||_\infty, \tag{12}$$

for some Lagrange multiplier λ of (10). Interestingly, explicit knowledge of the Lagrange multipliers to obtain an upper bound on the value of ϵ can be avoided. In fact, according to [19, Proposition 5.2], (12) is implied by

$$\epsilon < \frac{1}{2 \max_{\Delta P \in \mathcal{F}} \|\nabla f(\Delta P)\|_\infty}$$

where $\mathcal{F}$ is the feasibility set of (10). Note that (11) is always feasible, which enables us to consider just one objective function for the entire regulation period, rather than having one as the aggregate microgrid cost function when (10) is feasible and the other as the difference between the required and the procured regulation when it is infeasible.

To solve problem (11) in a distributed way, we notice that all constraints are local but the objective function f^ϵ does not have a separable structure. For simplicity of exposition, we ignore the local constraints from here on, albeit the ensuing treatment easily generalizes to the general situation. We can rewrite (11) as

$$\begin{aligned} \min_{\Delta P, u} \quad & f(\Delta P) + \frac{1}{\epsilon}[\Delta P_R - u]^+ \\ \text{s.t.} \quad & u = \sum_{\alpha=1}^{m} \Delta P_\alpha \end{aligned} \tag{13}$$

whose Lagrangian is given by

$$L(\Delta P, u, \lambda) = \sum_{\alpha=1}^{m} f_\alpha(\Delta P_\alpha) + \frac{1}{\epsilon}[\Delta P_R - u]^+ + \lambda(u - \sum_{\alpha=1}^{m} \Delta P_\alpha)$$

We propose the following algorithm to find the optimizers of (13)

$$\dot{\Delta P}_\alpha = -\nabla_\alpha L(\Delta P, u, \lambda) = -\nabla f_\alpha(\Delta P_\alpha) + \lambda \ \forall \alpha \tag{14a}$$

$$\dot{u} = -\nabla_u L(\Delta P, u, \lambda) = \left[\frac{1}{\epsilon}\right]^+_{\Delta P_R - u} - \lambda \tag{14b}$$

$$\dot{\lambda} = \nabla_\lambda L(\Delta P, u, \lambda) = u - \sum_{\alpha=1}^{m} \Delta P_\alpha \tag{14c}$$

The dynamics for u and λ can be executed by the RTO, which would be broadcasting the value of λ to the aggregators, and the dynamics for ΔP_α can be executed by the α-th aggregator. This makes sense as the RTO would be knowing the total regulation being provided from all the microgrids at all times. Also, the dynamics for the regulation power that each aggregator needs to provide requires the knowledge of only its cost function and λ. The following result, which we state here without proof due to reasons of space, characterizes the convergence properties of (14).

Proposition 5.1: (Asymptotic convergence to optimizers). The dynamics in (14) find the optimal solution of (13).

VI. Simulation

Here we illustrate the use of the proposed microgrid abstractions and the performance of the RTO-DERP coordination strategy. We consider 10 participating microgrids, each with 5 controllable nodes and different numbers of uncontrollable nodes ranging from 2 to 5. Regarding the topology of the power network inside each microgrid, we consider 5 graphs and instantiate each one twice with different parameter values (e.g., operating range and ramp rates of controllable nodes, values of loads, flow limits of lines). This way all microgrids are different from each other. We assume that the cost functions of the controllable nodes are quadratic with different coefficients. In the interest of space, we do not give the specific values of all the parameters here.

We use the dynamic regulation test signal available from the Pennsylvania-New Jersey-Maryland Interconnection (PJM) website [20]. We scale the signal by a factor of 4 and then clear the market for the required capacity and mileage. We provide the results of [CP1] in Table I. The signal updates the regulation set points every 2 seconds. We

Aggregator	Bids				Awards	
	Capacity (p.u.)	Capacity price ($\times$1000 \$/p.u.)	Mileage (p.u.)	Mileage price ($\times$1000 \$/p.u.)	Capacity (p.u.)	Mileage (p.u.)
1	1.20	0.84	8.00	0.20	1.20	5.82
2	1.20	1.17	14.76	0.40	0.28	0.28
3	0.60	0.20	4.60	0.23	0.60	0.60
4	1.10	3.38	10.00	0.25	0.00	0.00
5	1.30	4.82	15.08	0.87	0.00	0.00
6	1.60	1.35	13.02	0.57	0.00	0.00
7	0.30	0.58	2.25	0.30	0.30	0.30
8	0.70	0.60	4.00	0.10	0.70	4.00
9	0.40	1.13	0.94	0.10	0.40	0.94
10	0.50	1.00	1.53	0.10	0.50	1.53

TABLE I

INPUT AND OUTPUT OF THE MARKET CLEARING STAGE [CP1].

implement the dynamics for 100 such updates, with the initial power levels for the algorithm (14) at every round be the optimal allocation obtained at the previous round. Figure 1 compares our the result with current practice of regulation signal allocation (cf. Section III). The performance is similar in terms of tracking: in fact, the sum of absolute differences between the procured regulation and required regulation is 5.50 p.u. with our approach versus 5.72 p.u. with current practice. In terms of cost, our proposed approach with cost 4.28 $ outperforms the current practice with cost 6.57 $.

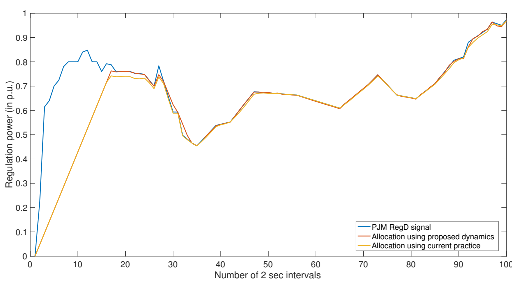

Fig. 1. Performance of the proposed dynamics and the current practice followed tested against first 100 updates of the PJM RegD signal.

VII. CONCLUSIONS

We have considered the problem of microgrid participation in the frequency regulation market. After identifying the limitations in current practice regarding the incorporation of cost of generation and operational limits in the disaggregation of the regulation signal, we have addressed two problems to enable the participation of microgrids. On the one hand, we have developed abstractions for the capacity, cost of generation, and ramp rates by carefully considering the combination of the individual resources inside the microgrid. This provides enough information for the microgrids to participate in the market clearance stage. On the other hand, we have employed these abstractions to design a provably correct algorithm that solves the RTO-DERP coordination problem to disaggregate efficiently the regulation signal. Future work will extend our work to microgrids with more general topologies, incorporate dynamic models for the behavior of individual DERs, investigate accelerated methods to improve the speed of convergence, and explore the design and domain of applicability of fully distributed algorithmic solutions to the RTO-DERP coordination problem.

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